

Balanced Numeracy

Grades 3 - 7



April 20th 10:30 - 12:00 p.m.
Prince George, BC

Acknowledgement



We gratefully acknowledge we are here today to learn on the unneeded shared traditional territories of the Lheidl T'enneh First Nations.

Learning Intentions



- I understand the key components that comprise a Balanced Numeracy Program, how they fit together (e.g., whole class, small group, and independent practice) and what this could look like in the classroom.
- I understand learning is developmental. I have some ideas I can use to identify what my students know and their next steps.
- I have an emerging understanding of what Balanced Numeracy could look like over a week.
- I am aware of various resources that I can use to support my Balanced Numeracy Program (e.g., books, websites)

Balanced Numeracy Network

Balanced numeracy is a framework that incorporates a diverse range of organizational structures, assessments and instructional practices that are intentional and responsive to students and curriculum. Balanced numeracy provides opportunities for students to uncover, construct, and apply mathematical understandings.



<https://sites.google.com/view/bcnumeracynetwork/home>

Balanced Literacy (K to 12)	Balanced Numeracy (K to 12)
<p>Routines:</p> <ul style="list-style-type: none"> Transitions (Question of the Day; Morning Message, 'soft start', provocations, etc.) <p>Flexible Groupings:</p> <ul style="list-style-type: none"> Whole class (read aloud, mini-lessons, etc.) Small groups (mini-lessons, guided reading, literacy centers, Literature Circles, etc.) Individual (conferences & interviews, authentic practice, student choice) <p>Explicit and Intentional Planning</p> <p>Gradual Release of Responsibility</p> <ul style="list-style-type: none"> Modelling of reading, writing – 'think aloud' Shared reading and writing – 'think-pair-share' Guided reading and writing Independent and authentic practice in reading, writing, reflecting Individual choice <p>Reading & Writing Workshop (Structure)</p> <ul style="list-style-type: none"> Mini lessons Lots of time reading, writing, thinking Lots of descriptive, individual feedback <p>Rich Learning Environment</p> <ul style="list-style-type: none"> Classroom library, literacy centers, read aloud, word wall, anchor charts for routines and criteria Place-based learning, outdoor learning, First Peoples Principles 	<p>Routines:</p> <ul style="list-style-type: none"> Daily math investigations Transitions (provocations, number sense routine, etc.) <p>Flexible Groupings:</p> <ul style="list-style-type: none"> Whole class (modelling, "Number Talks", mini-lessons, "Math Makes Sense: Explore") Small group (guided math, mini-lessons, math centers, collaborative tasks and problem solving) Individual (practice, conferences, interviews) <p>Explicit and Intentional Planning</p> <ul style="list-style-type: none"> Model – think aloud Guided small group practice Independent exploration and practice <p>Math Workshop (Structure)</p> <ul style="list-style-type: none"> Small group Conferencing with teacher Teacher prompting and questioning to encourage further learning <p>Rich Learning Environment</p> <ul style="list-style-type: none"> Math community, student discourse Choice, play, inquiry, joy, fun Access to materials (manipulatives, etc.) Word wall, anchor charts, visuals Rich tasks, problem-solving, inquiry Visual tools (visualize, spatial reasoning)

Using what we knew about Balanced Literacy, we asked ourselves "What does this mean for Numeracy?"

<p>Foundations</p> <ul style="list-style-type: none"> Reading and writing strategies Phonological awareness, phonics, sight words, etc. 	<p>Foundations</p> <ul style="list-style-type: none"> Conceptual connections Computational fluency Concrete, pictorial, abstract – at all grade/age levels (connected to formal introduction of a concept) Common language – vocabulary Compose and decompose quantities, shapes, and fractions Strategies that demonstrate flexible thinking
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What key parts are needed?

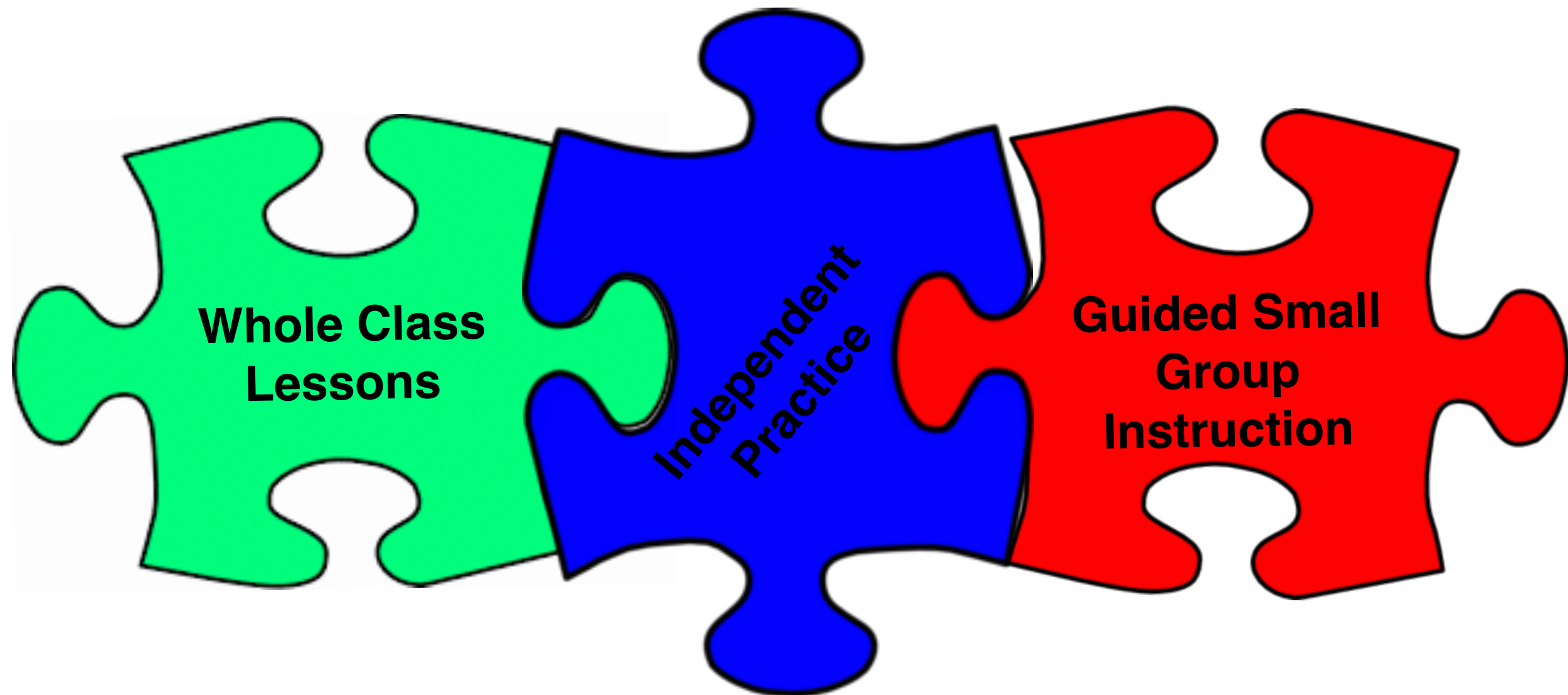


**Whole Class
Lessons**

**Independent
Practice**

**Guided Small
Group
Instruction**

What key parts are needed?

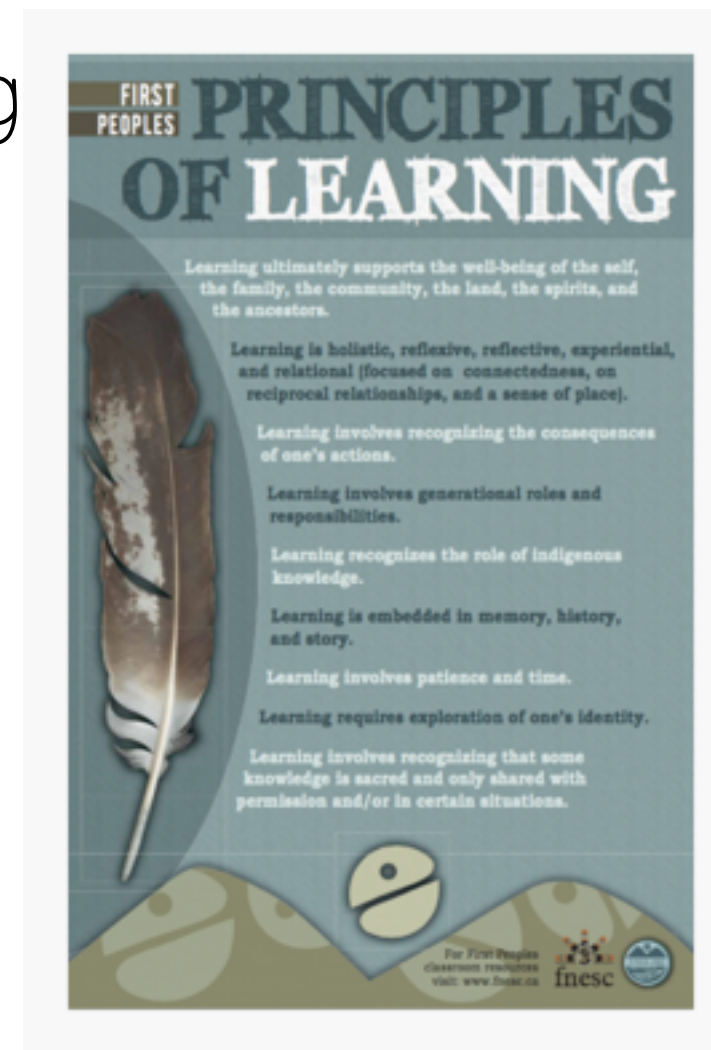


Creating rich math learning contexts involves not only planning the activities that students will complete, but also considering the kinds of interactions, language use, and critical thinking opportunities in which we want our students to engage.

– Cathy Marks Krpan (2018)

What beliefs about children and learning guide our work?

- First Peoples Principles;
- students learn to think like Mathematicians by being immersed in classrooms that foster the “Habits of Minds/Doing” of Mathematics;
- problem solving is fundamental to the study of mathematics;
- an inquiry-based approach includes rich tasks and student problem posing, which nurtures engagement, curiosity and deep understanding
- there is a progression of learning in mathematics.
- ALL students can do Math!



Whole Class Lessons

Learning opportunities that enhance student understanding of mathematics through whole class instruction.

Guiding Questions:

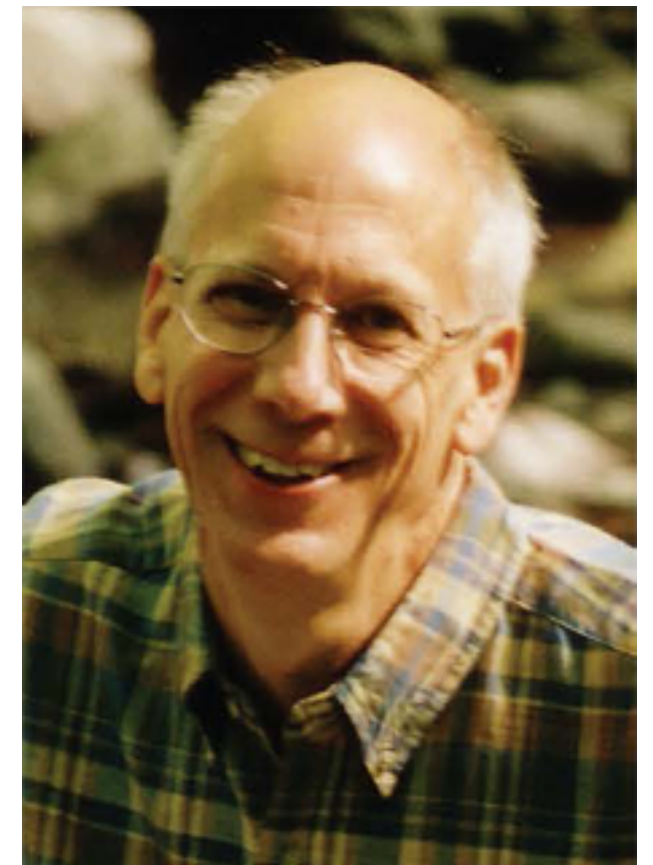
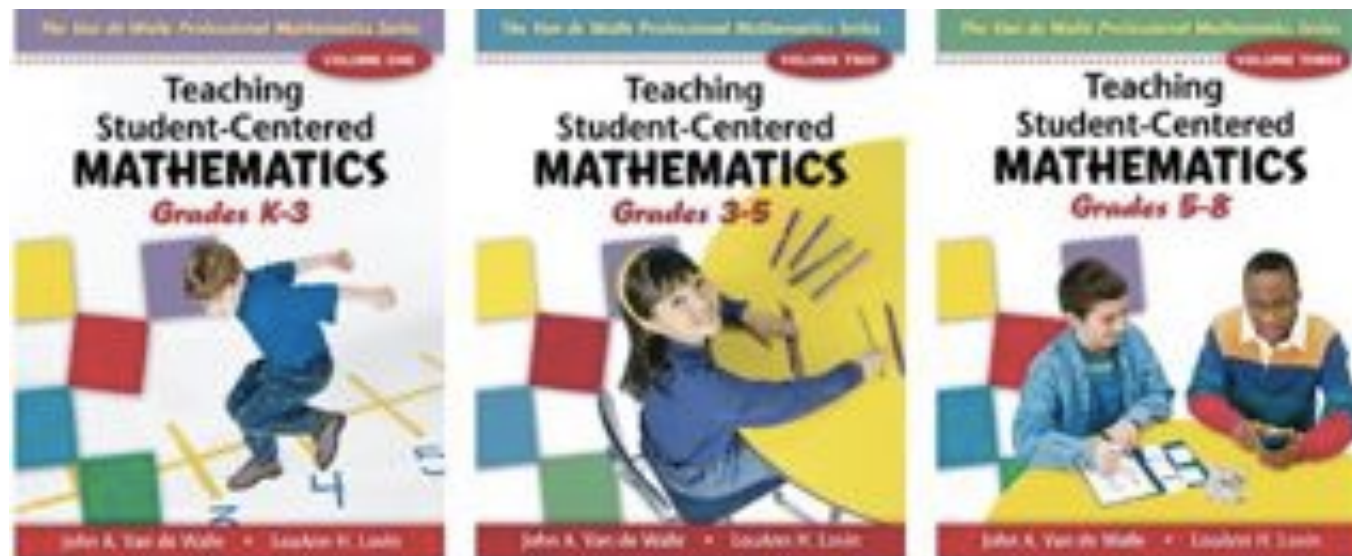
- How can I use Rich Routines to develop a community of learners in which students feel safe to take risks and make mistakes?
How will I use routines to provide opportunities to revisit concepts and model 'thinking aloud'?
- How will I activate prior knowledge?
- How will I find out what my students' know?
- What mathematical vocabulary needs to be introduced?
- Is there any available literature that will help students to make connections and provide visual connections to concepts?
- What learning opportunities can I use that will provide an entry point for ALL students?

Three Part Lesson Format

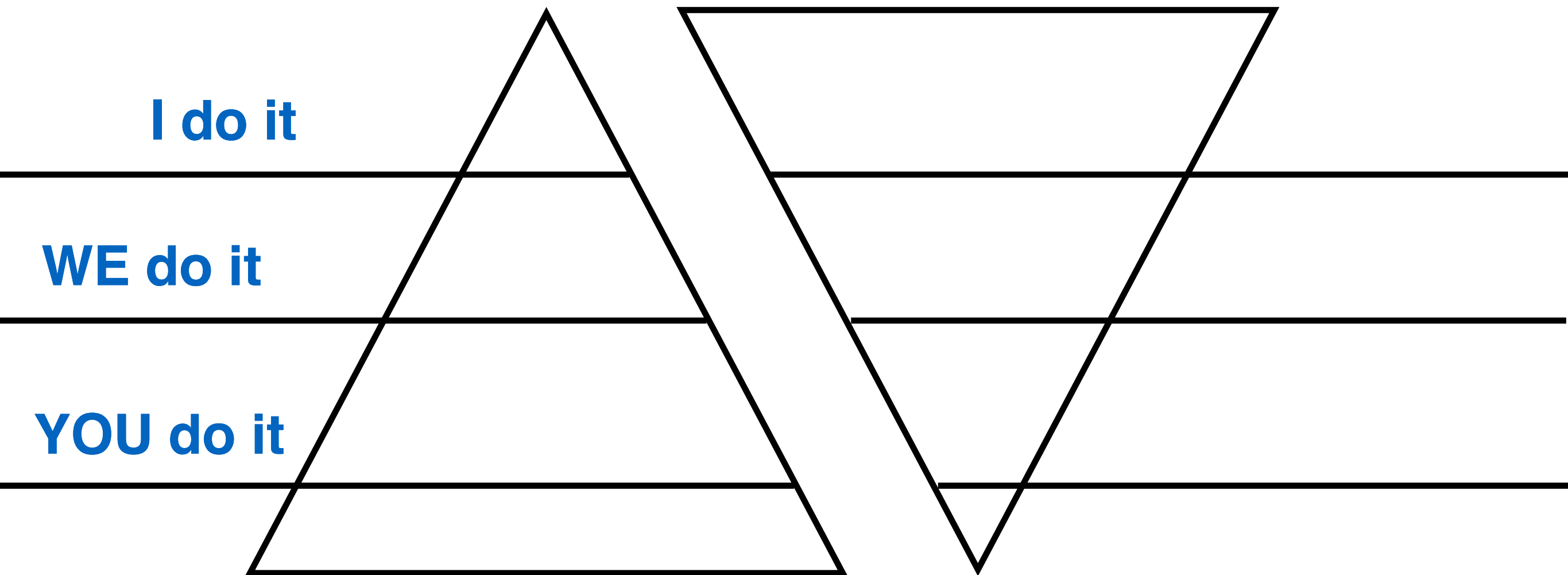
BEFORE: Engage your students – Activate prior knowledge –
Clear learning intentions – Pose the Problem

DURING: Exploration/Conferencing/Formative Assessment

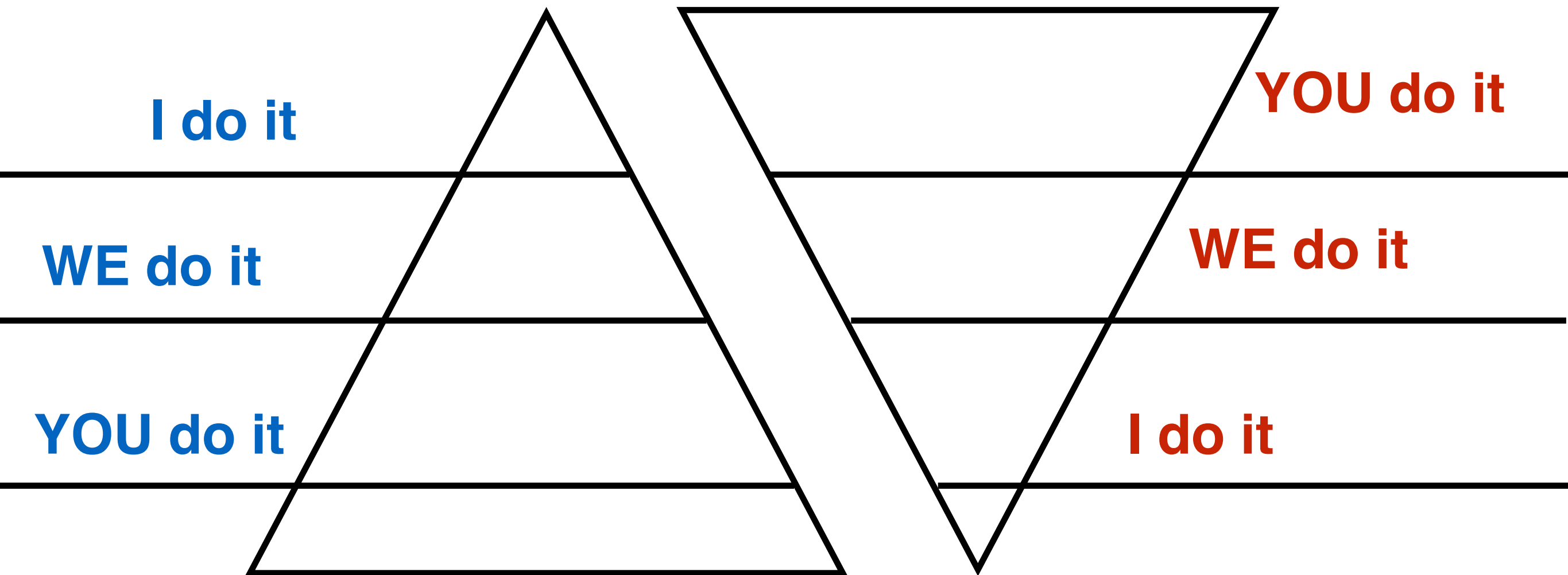
AFTER: Sharing – Connecting and Reflecting



Gradual Release Flipped

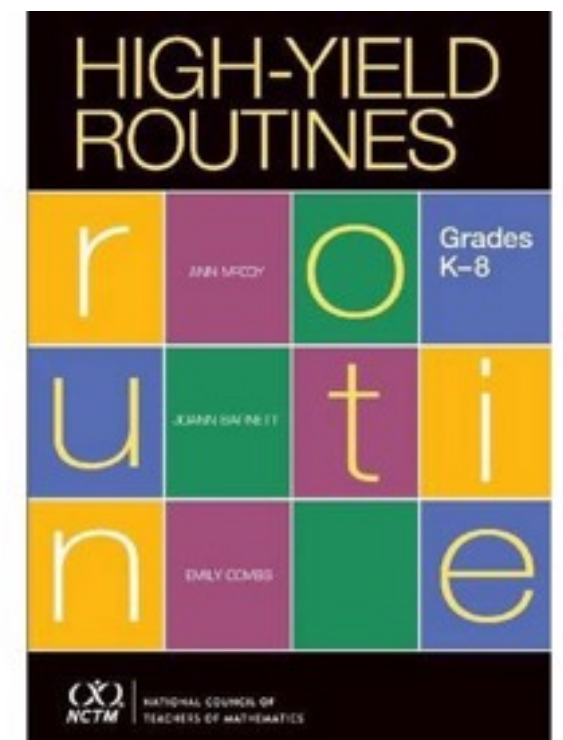
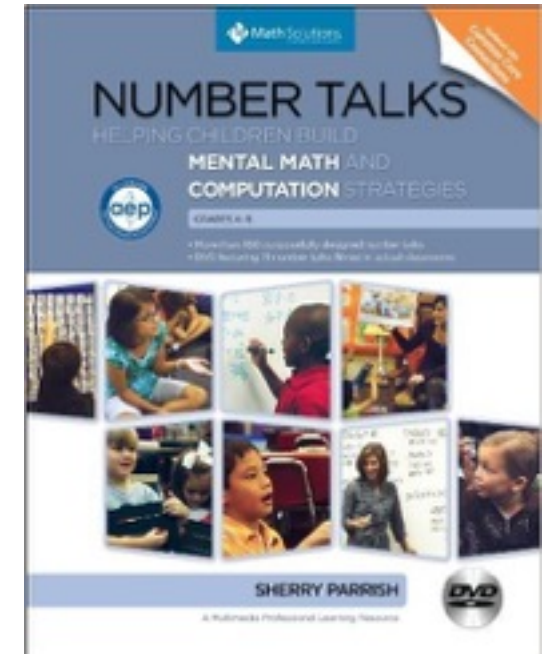
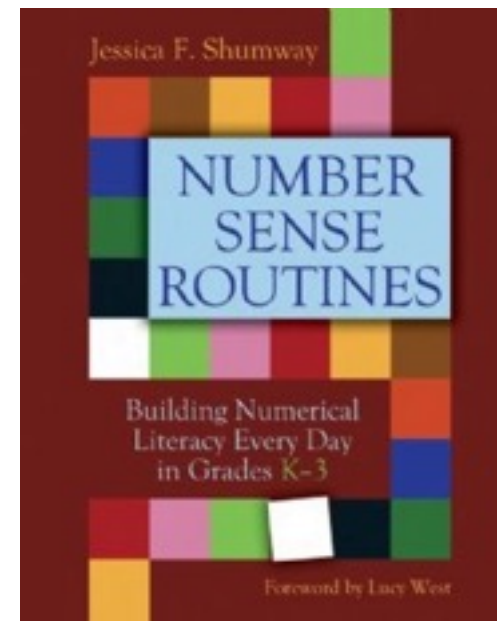


Gradual Release Flipped



What are Mathematical Instructional Routines?

- Collection of quick, low-prep 5 to 10 minute activities.
- They focus on the big ideas in Mathematics.
- They serve to reteach, reinforce, and enrich.
- Can be used as warm ups, mini lesson, with the whole class or in small groups.



Why use Number Routines?

- Builds a Math community where students feel safe to take risks and can learn from one and other
- Provides daily number sense experiences where students clarify their thinking, consider and test strategies, and build a repertoire of efficient strategies
- Fosters discussion about numbers and their relationships
- Responsive to students' understandings
- Allows for spiralling through concepts and helps students make connections to the big ideas in mathematics
- Emphasizes the core and curricular competencies in relation to mathematical content.

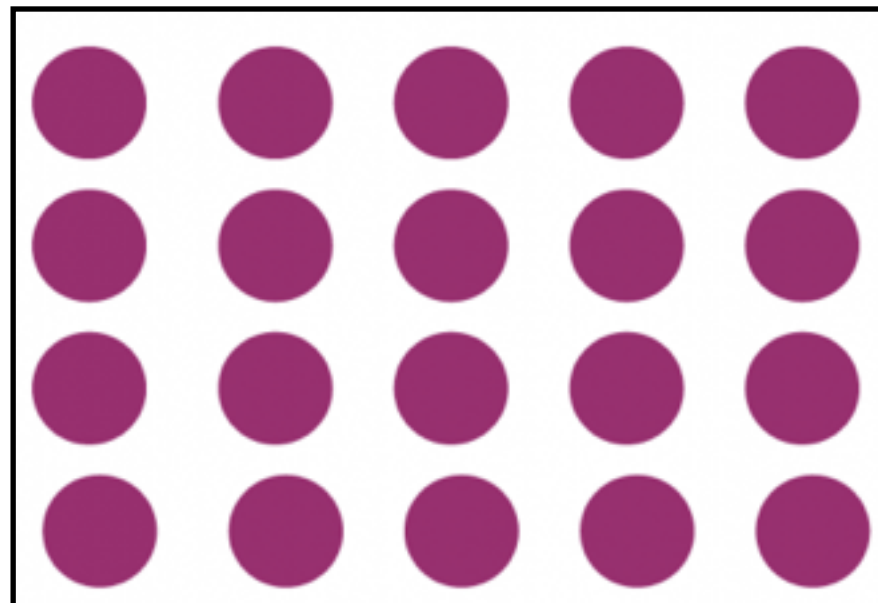
What Curricular Competencies are fostered?

- Reasoning and Analyzing through estimating and developing mental math strategies and abilities to make sense of quantities
- Understanding and Solving through using multiple strategies
- Communicating and Representing their thinking not only orally but through concrete materials, pictorial representations, and symbolically
- Connecting and Reflecting through visualizing and describing mathematical concepts, connecting mathematical concepts, and sharing and reflecting upon their thinking

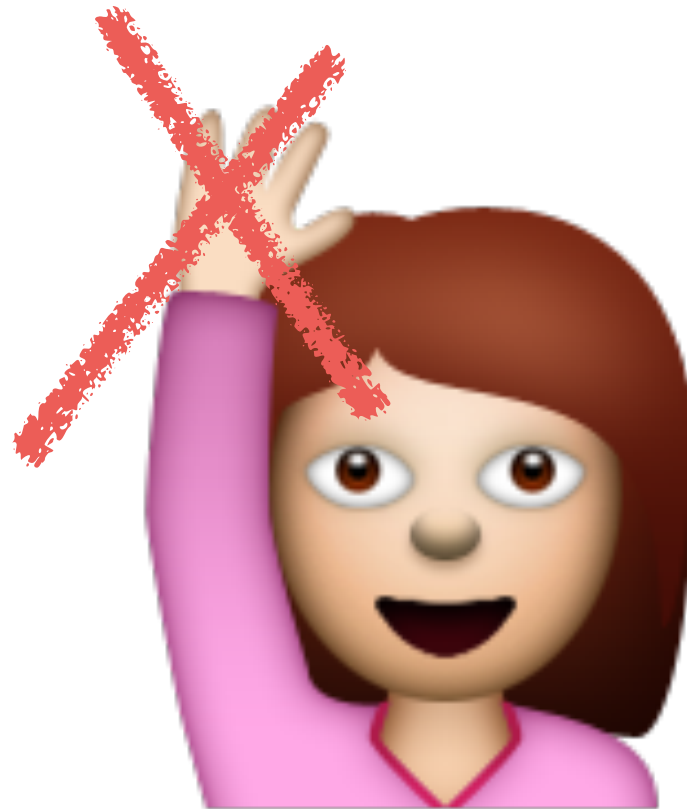
Number Talks Using Quick Images

Learning Intentions:

- develop multiple strategies for Decomposing (Mental Math)
- developing flexibility through use of multiple strategies
- Computational Fluency
- Place Value



Thinking time is needed



NO HANDS UP

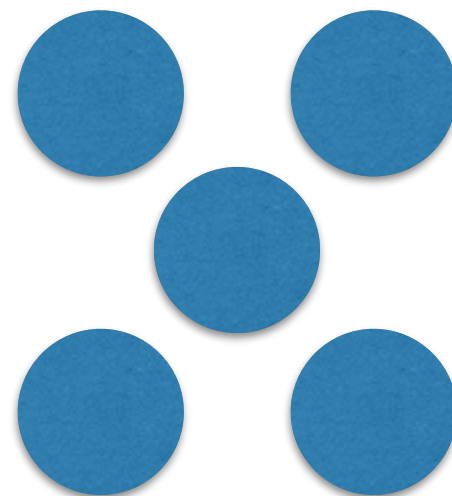
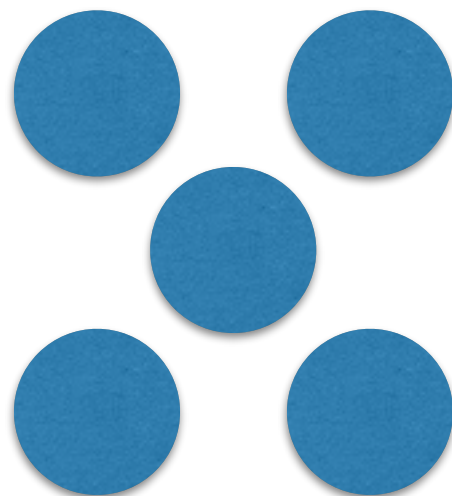
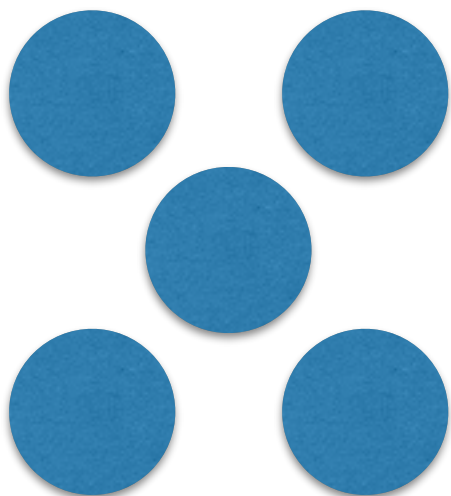
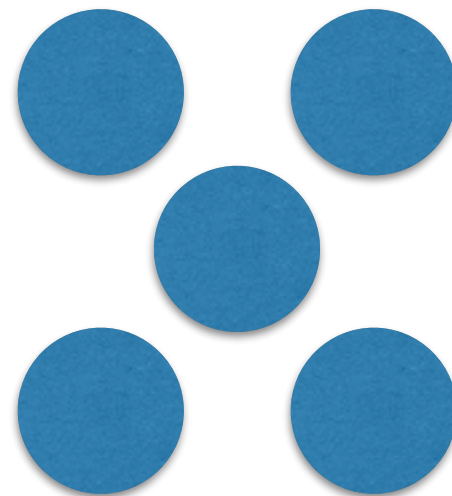
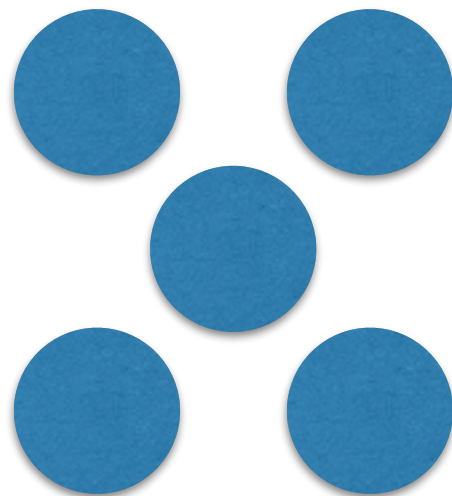
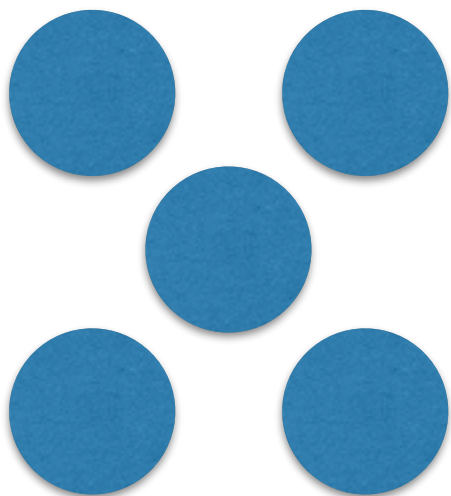
SECRET SIGNALS

Hold up one thumb if you have one way to find the answer.



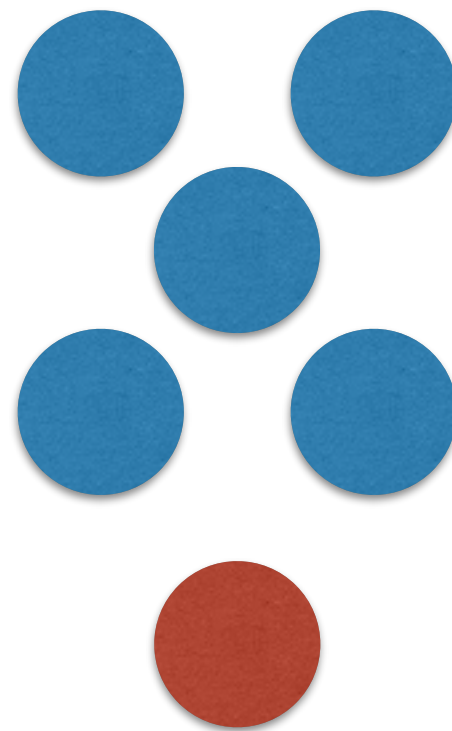
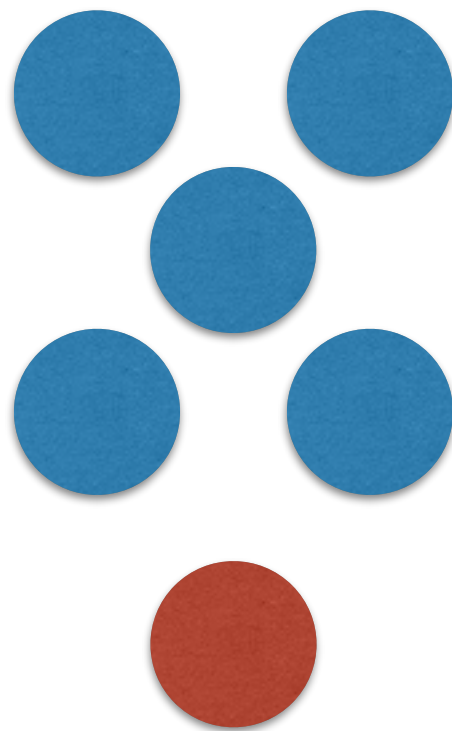
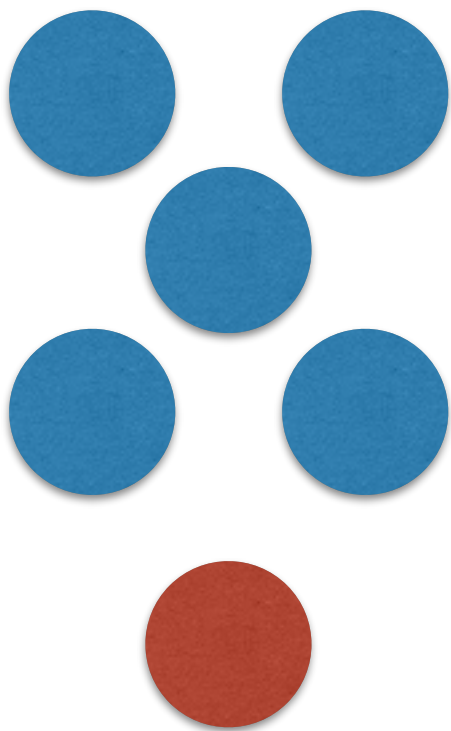
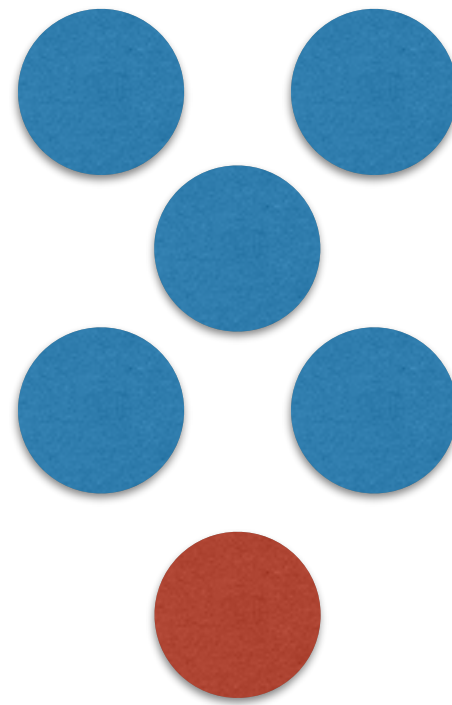
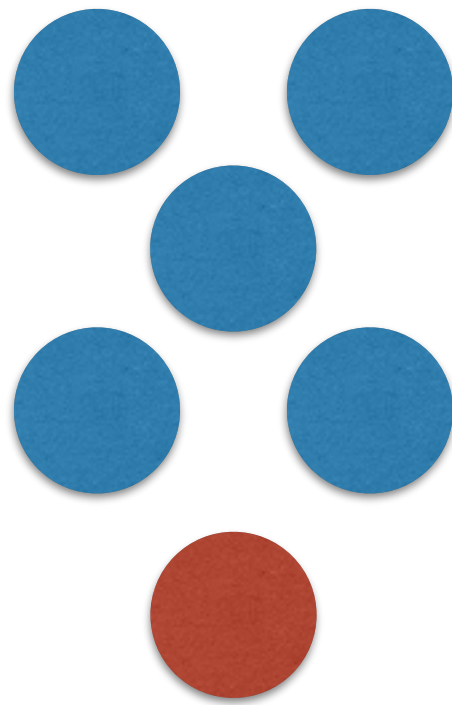
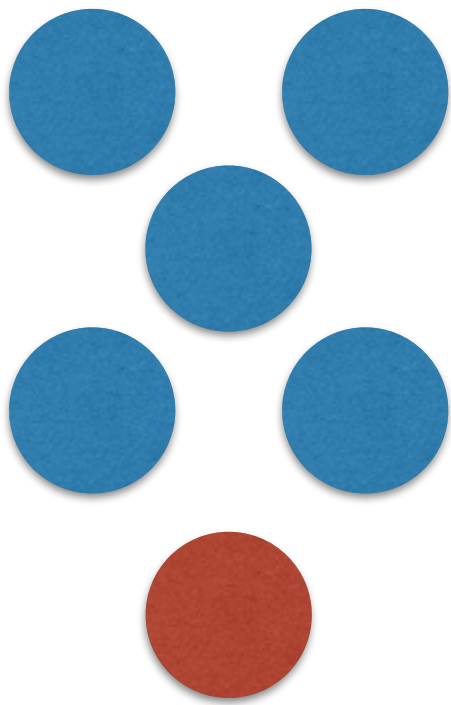
Hold up another finger if you another way...

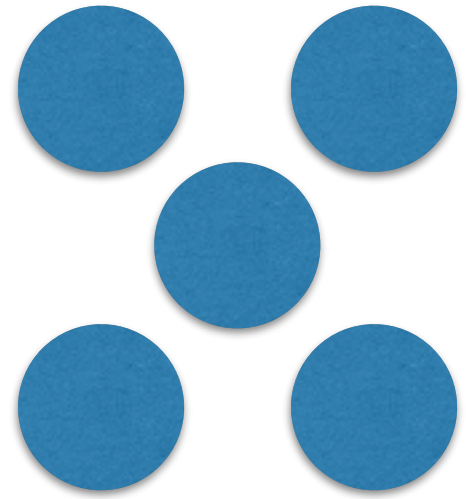
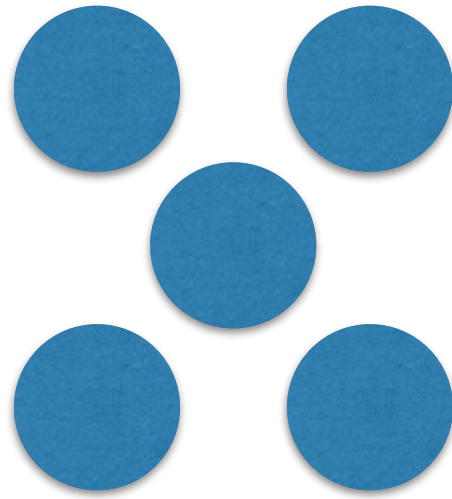
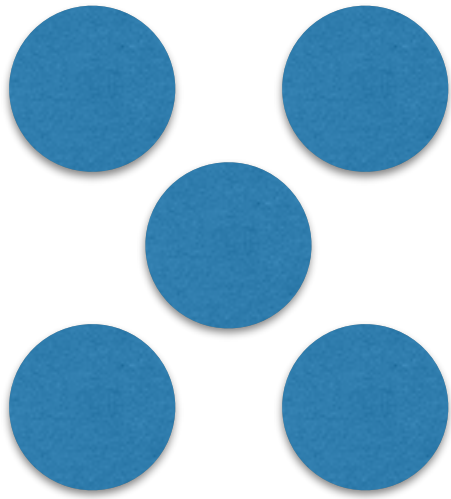
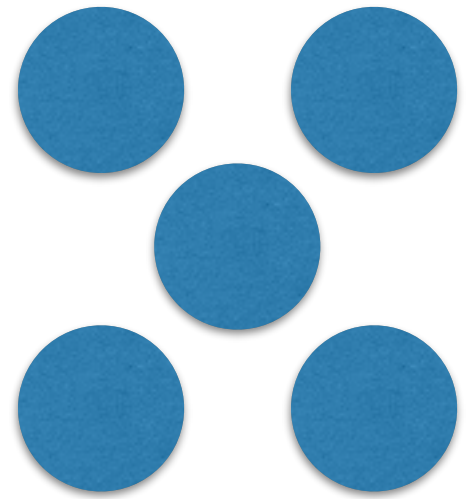
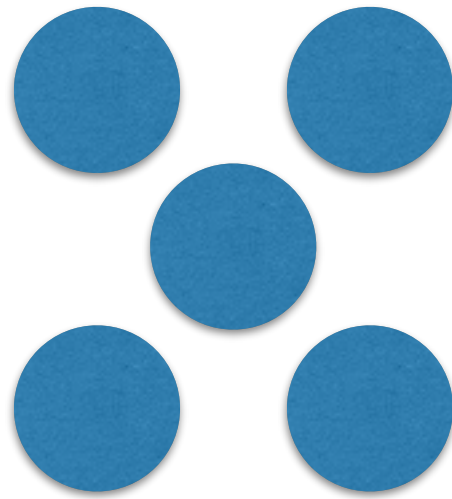
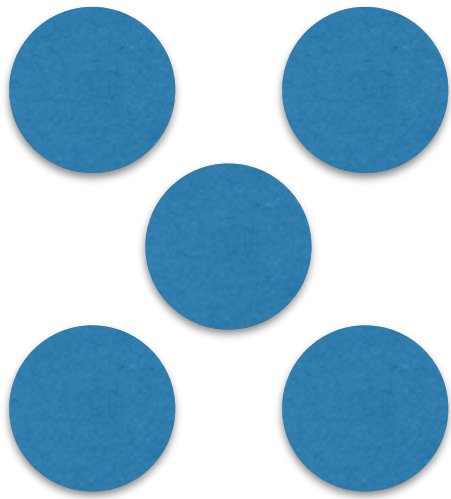




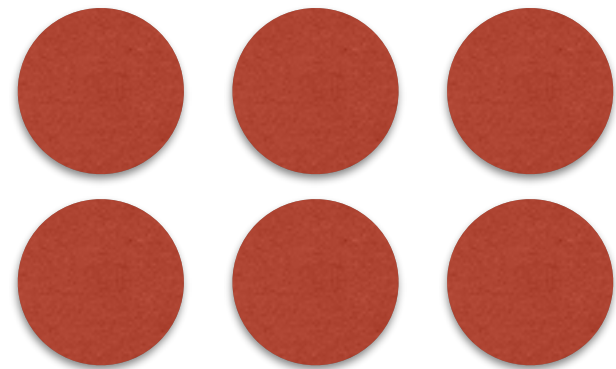
How many?

How did you see them?

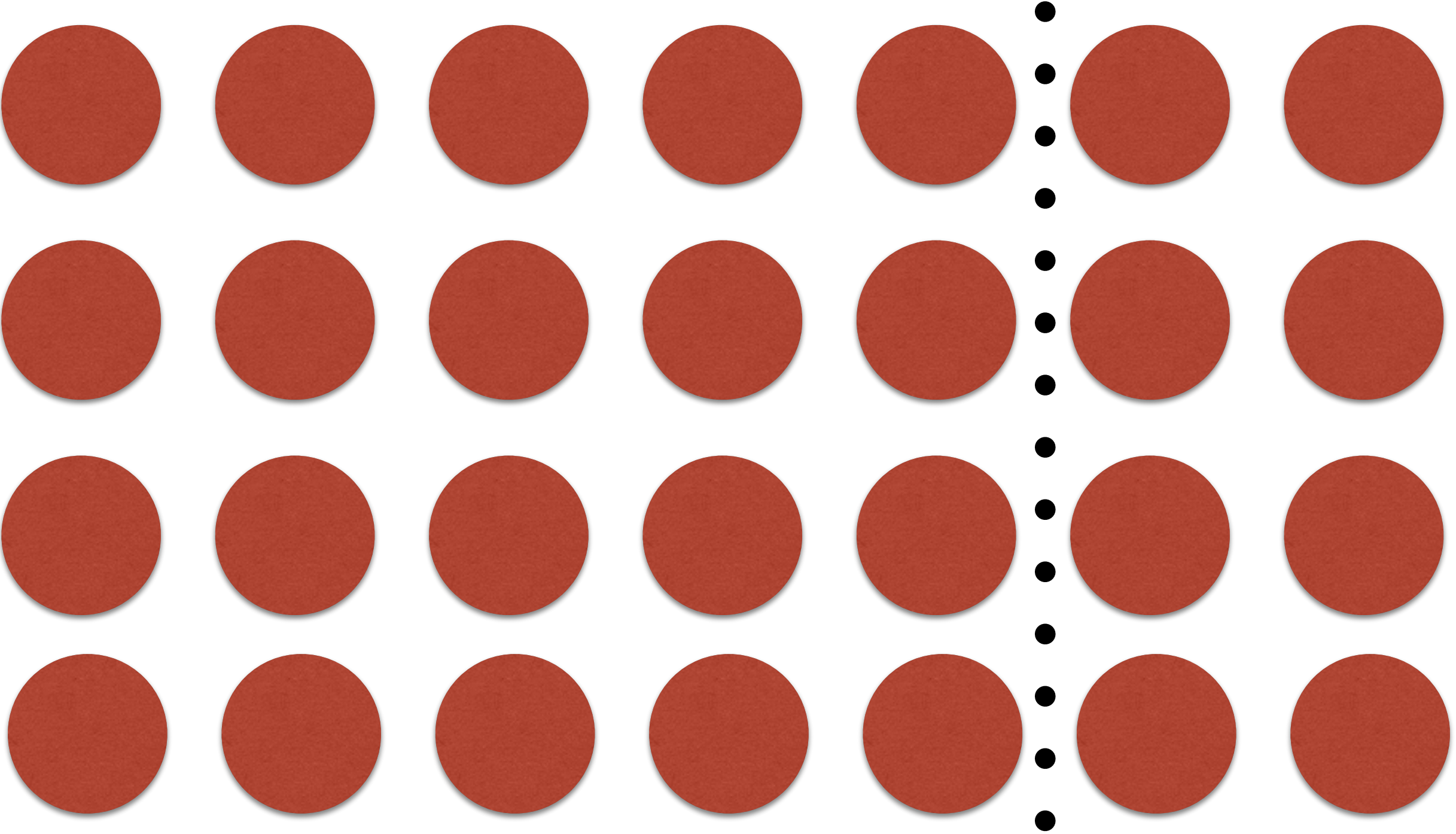




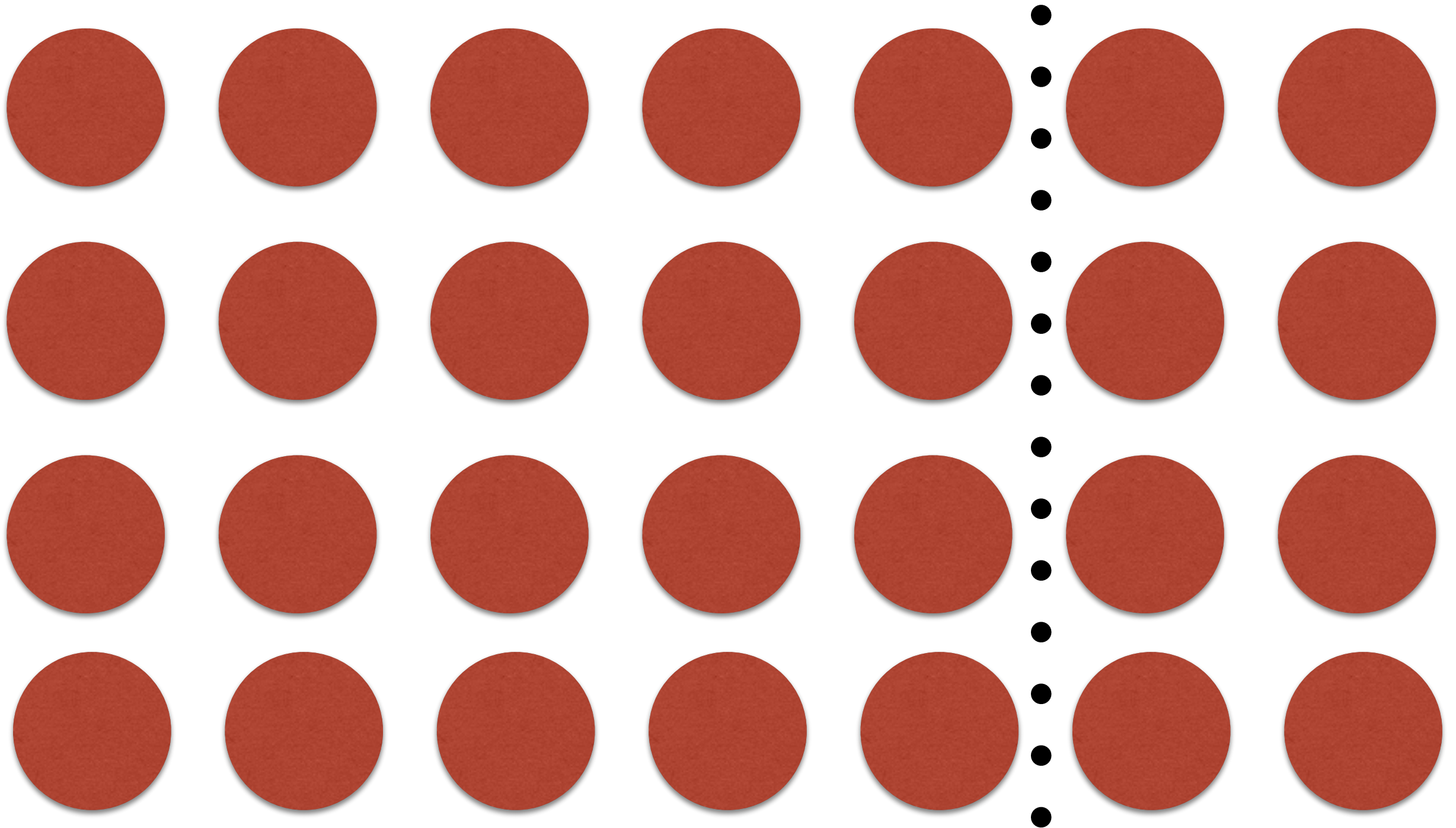
Partial Products

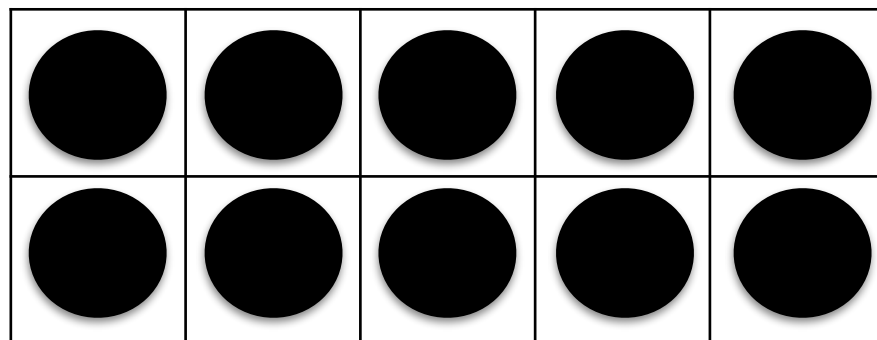
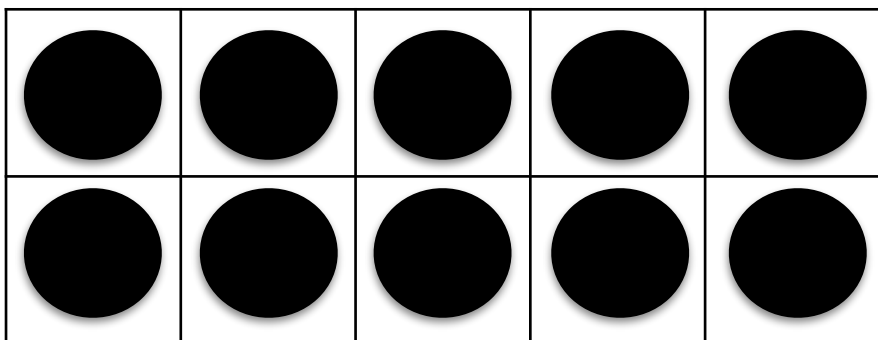
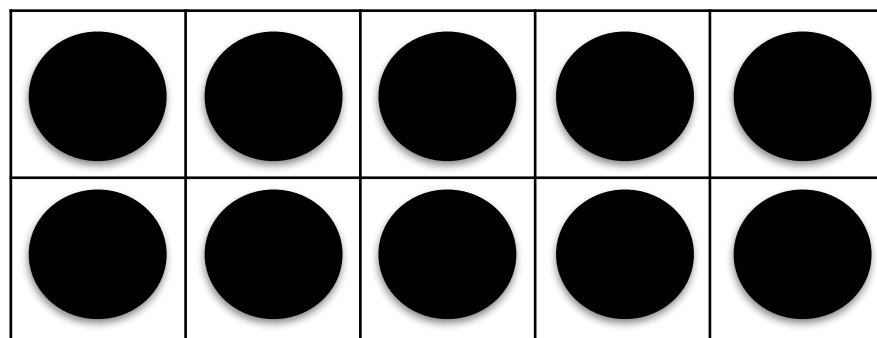
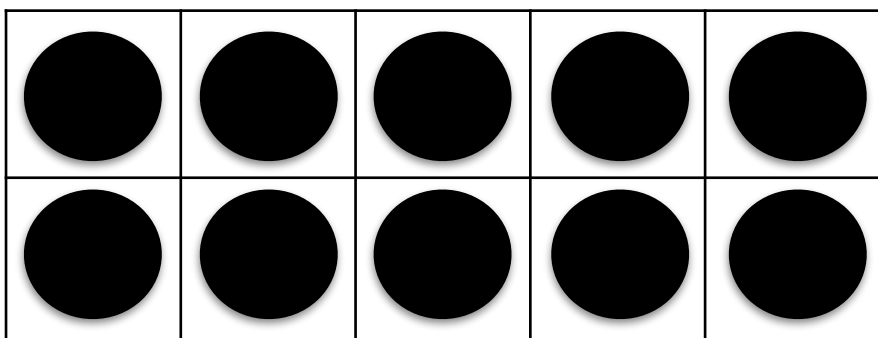
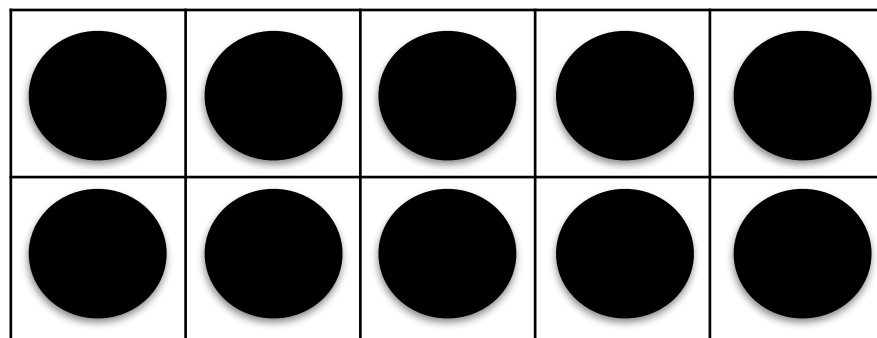
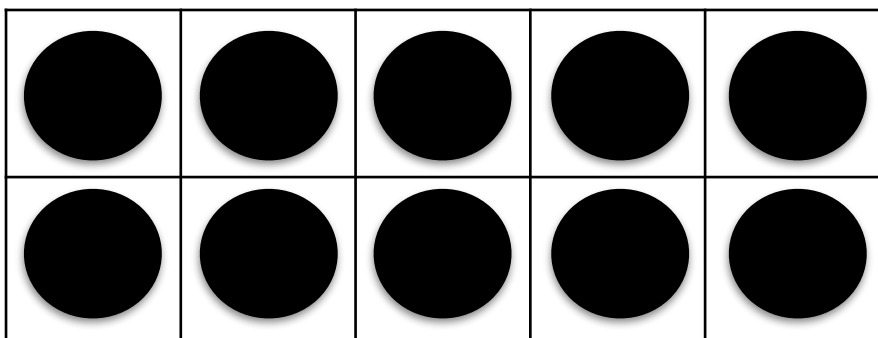
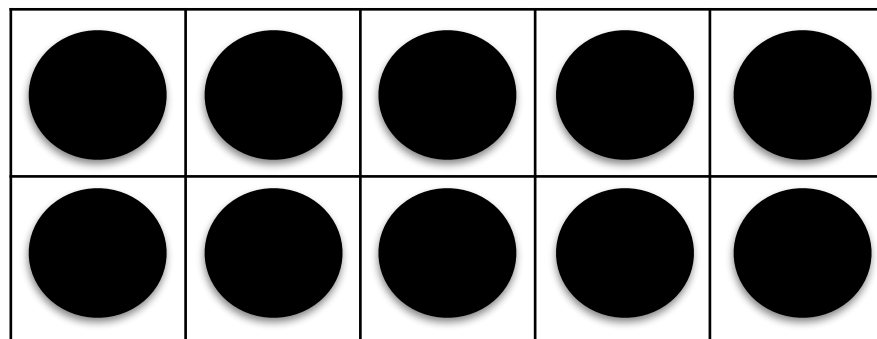
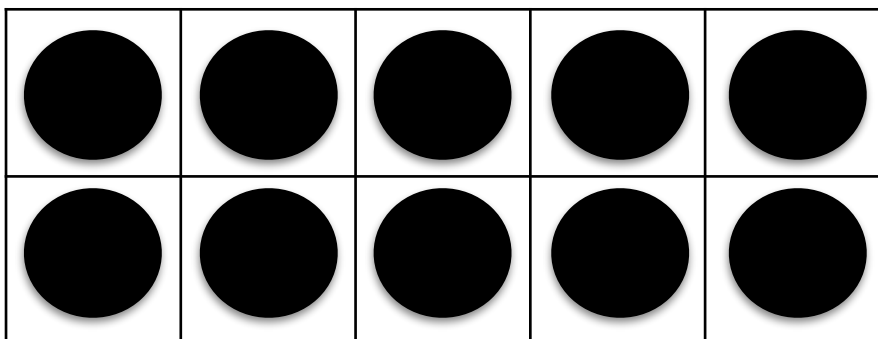


$$\begin{array}{c} 6 \\ \swarrow \quad \searrow \\ (6 \times 5) + (6 \times 1) \end{array}$$



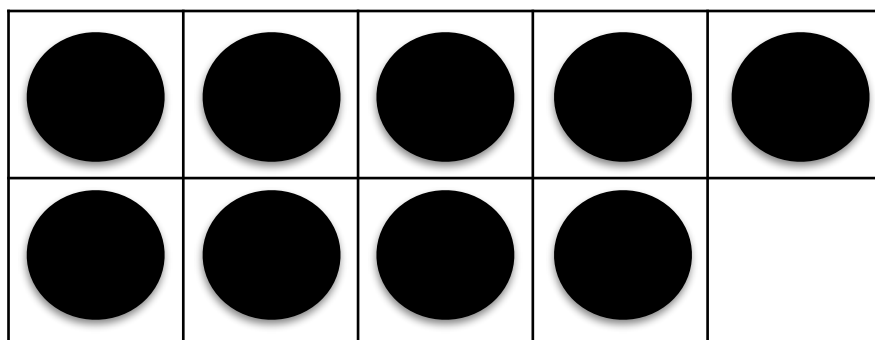
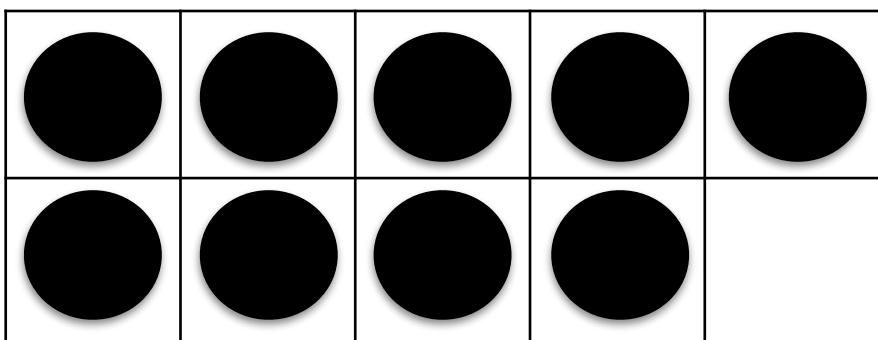
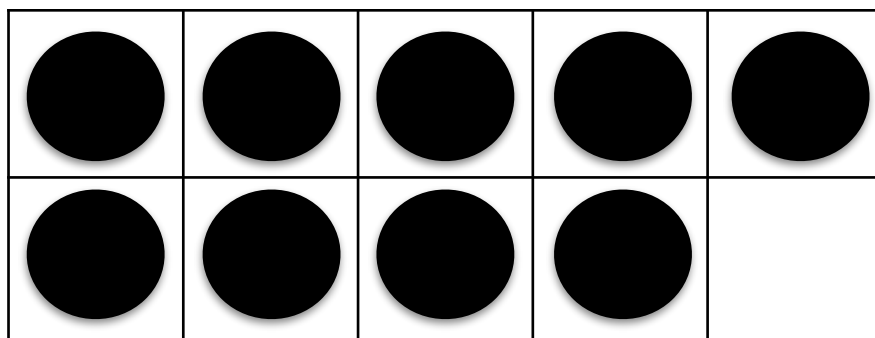
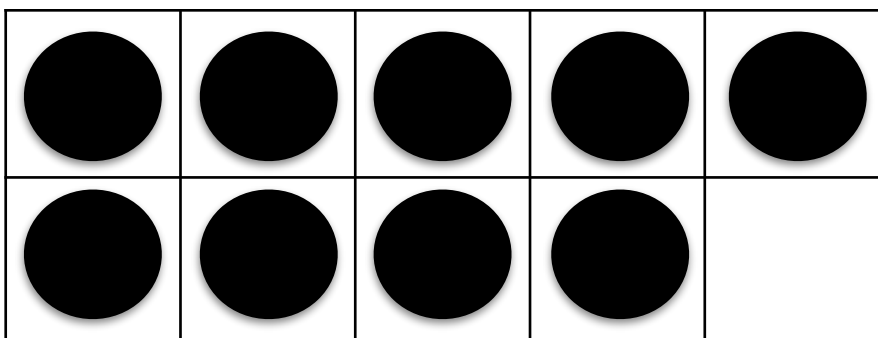
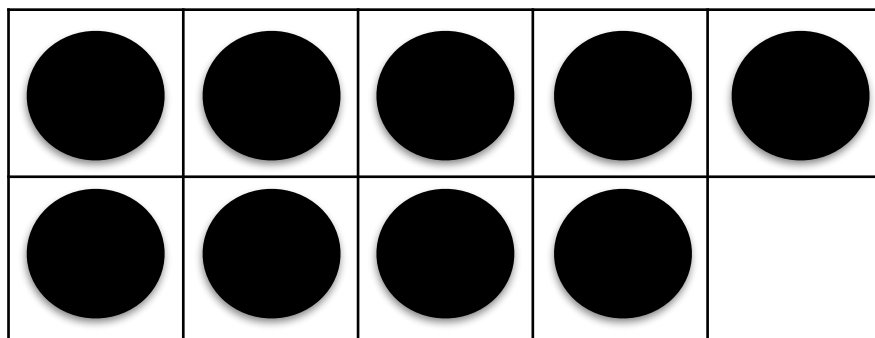
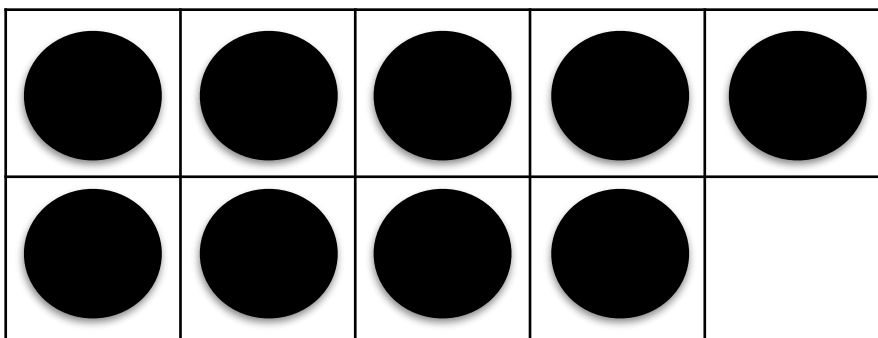
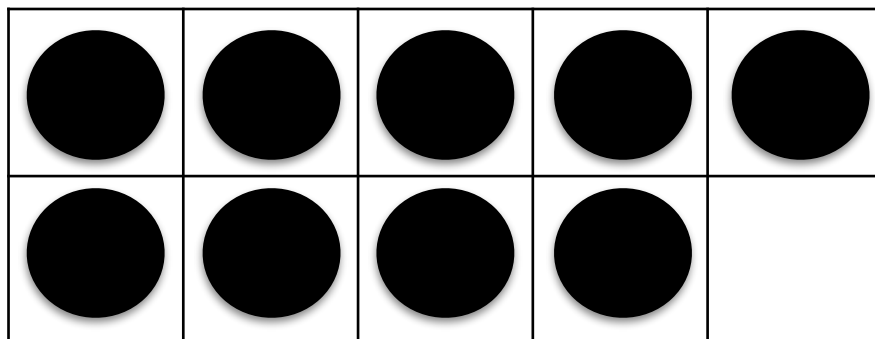
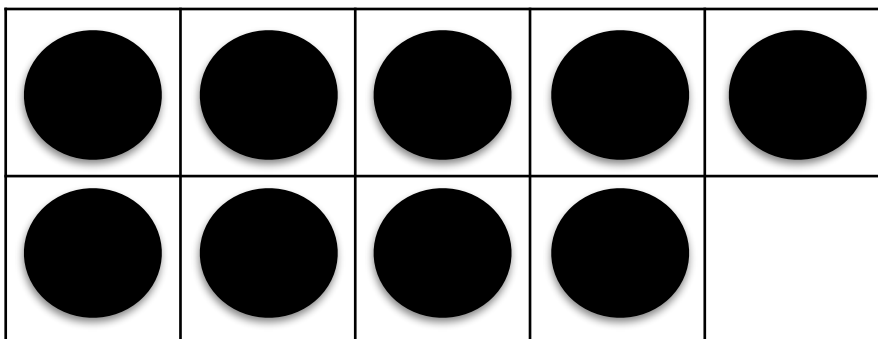
$$(4 \times 5) + (4 \times 2)$$

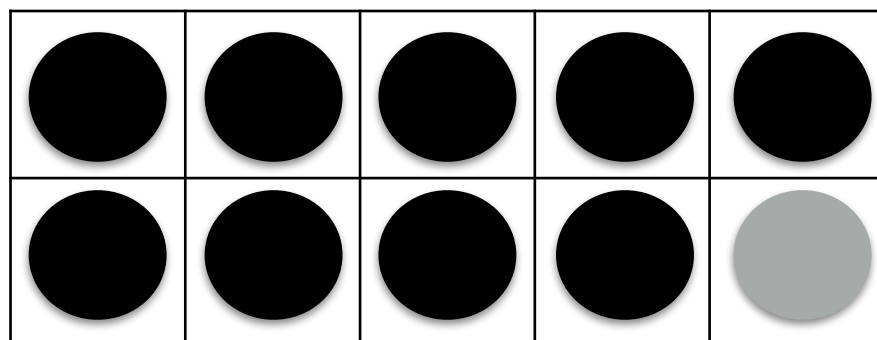
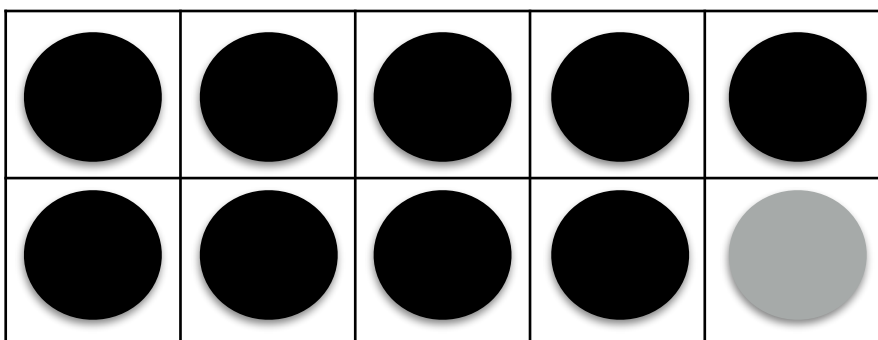
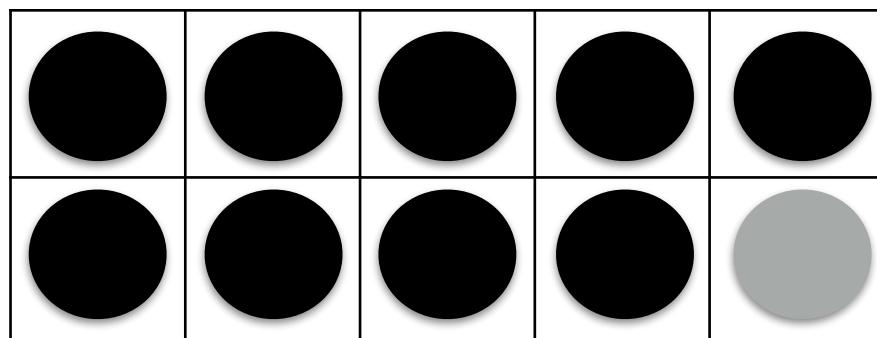
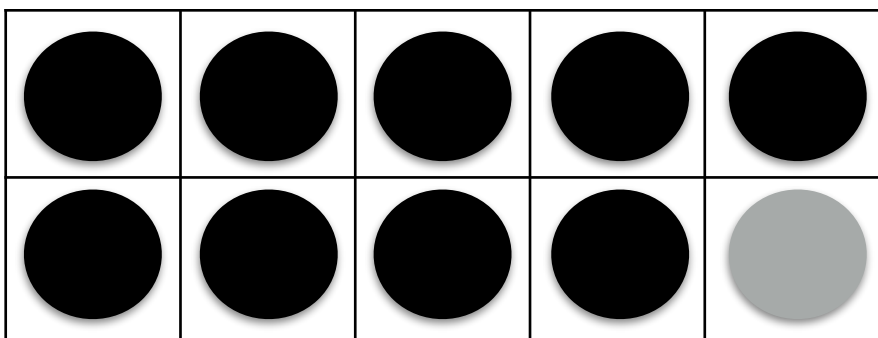
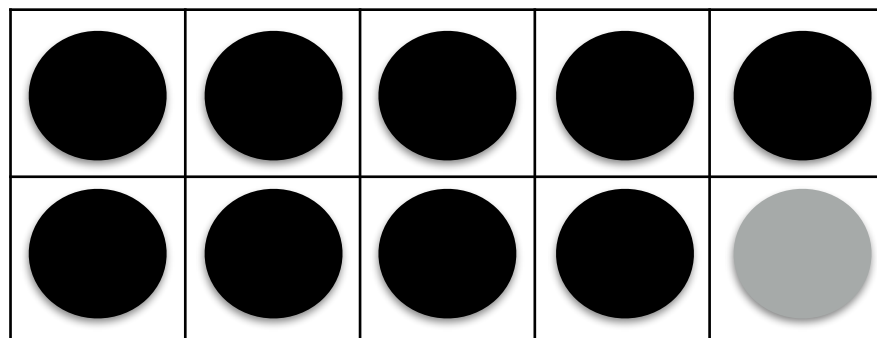
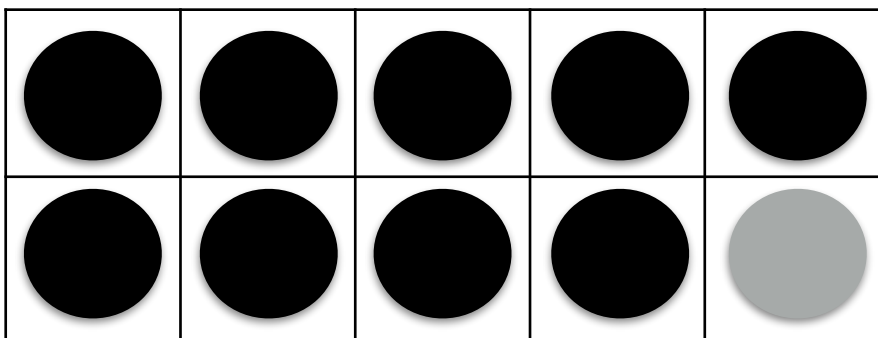
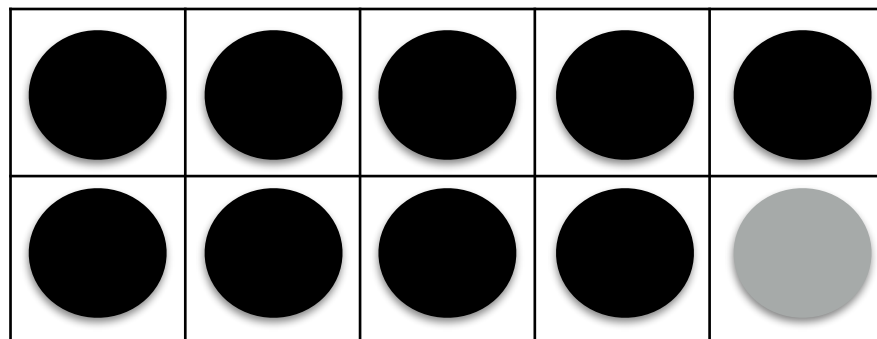
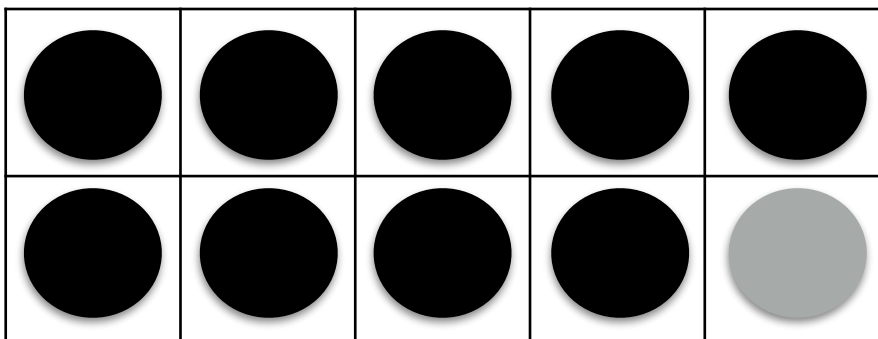


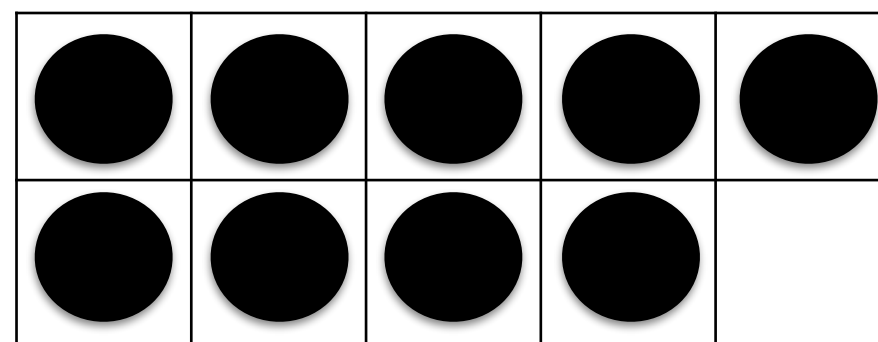
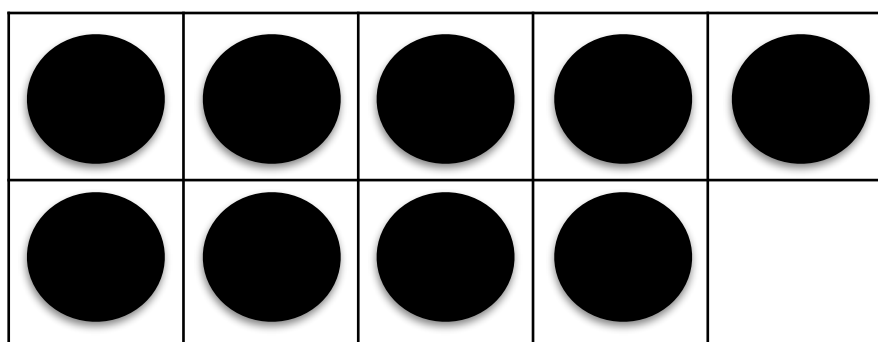
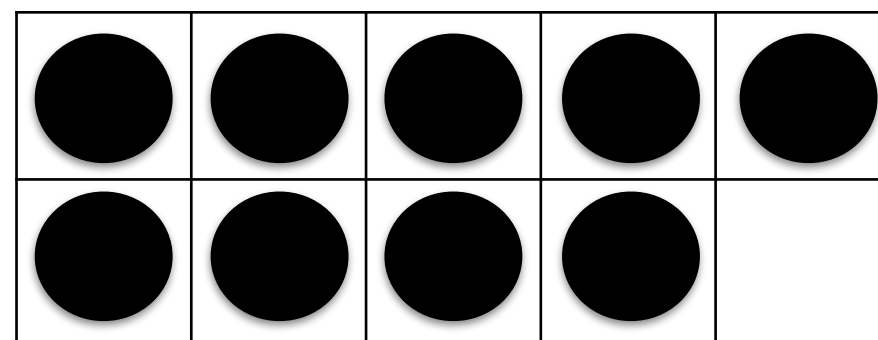
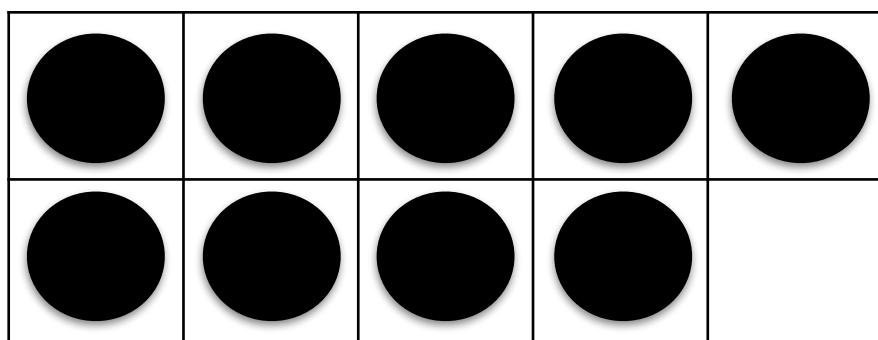
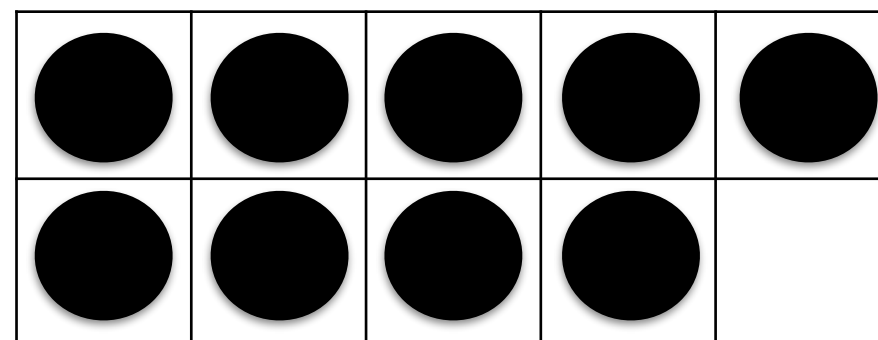
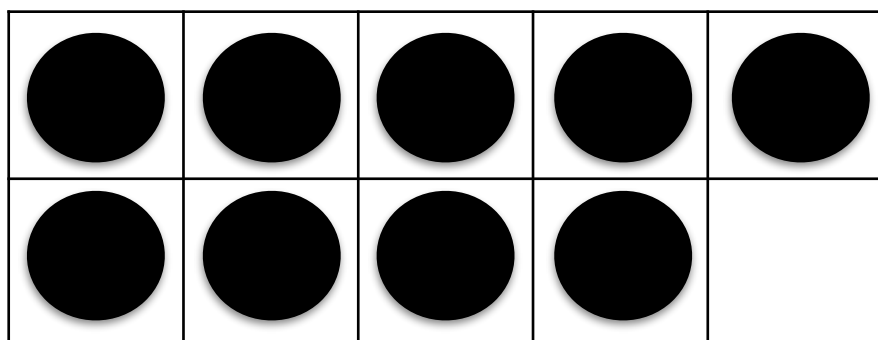
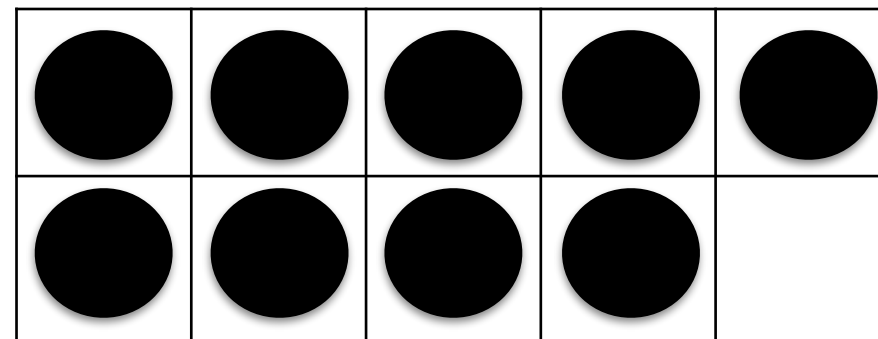
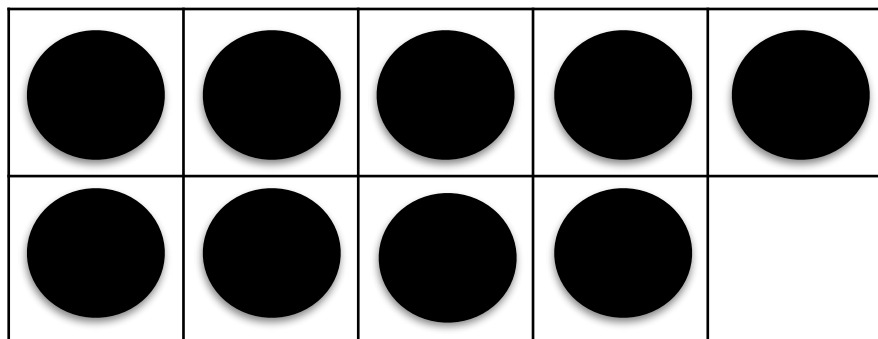


How many?

How did you see them?

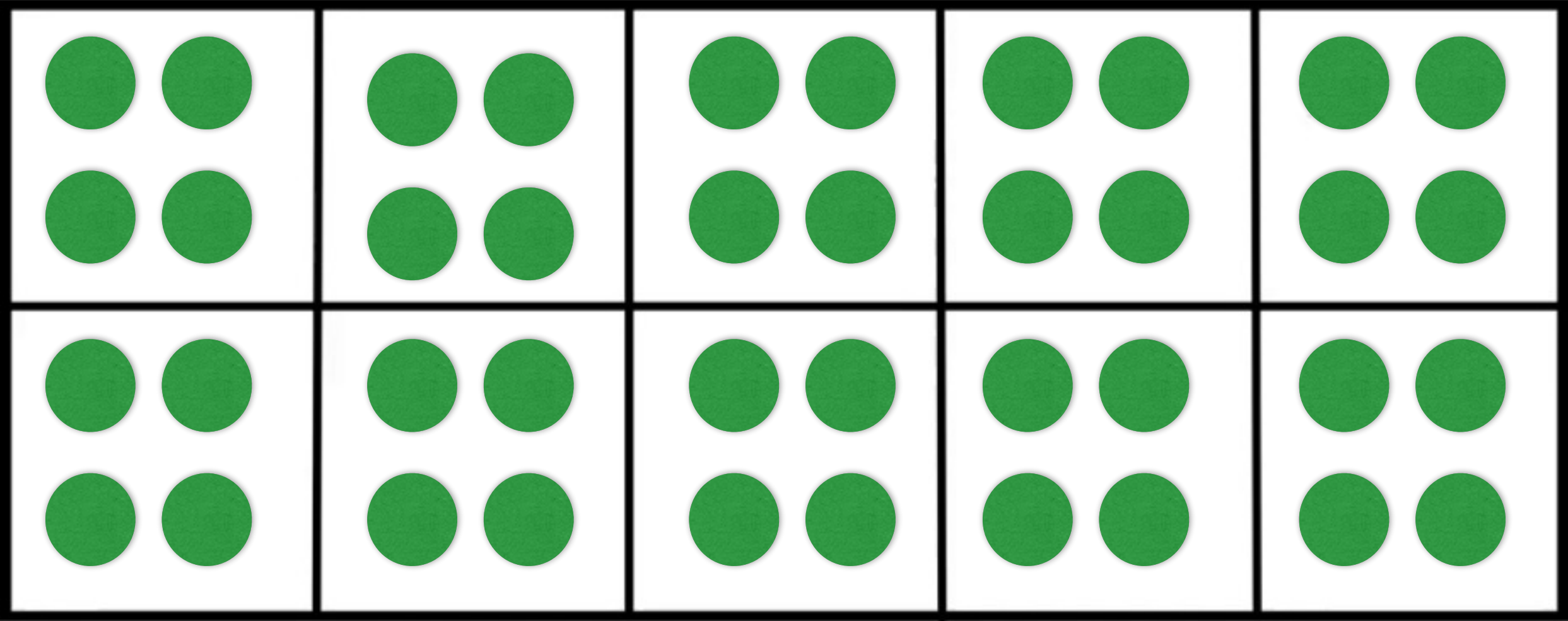






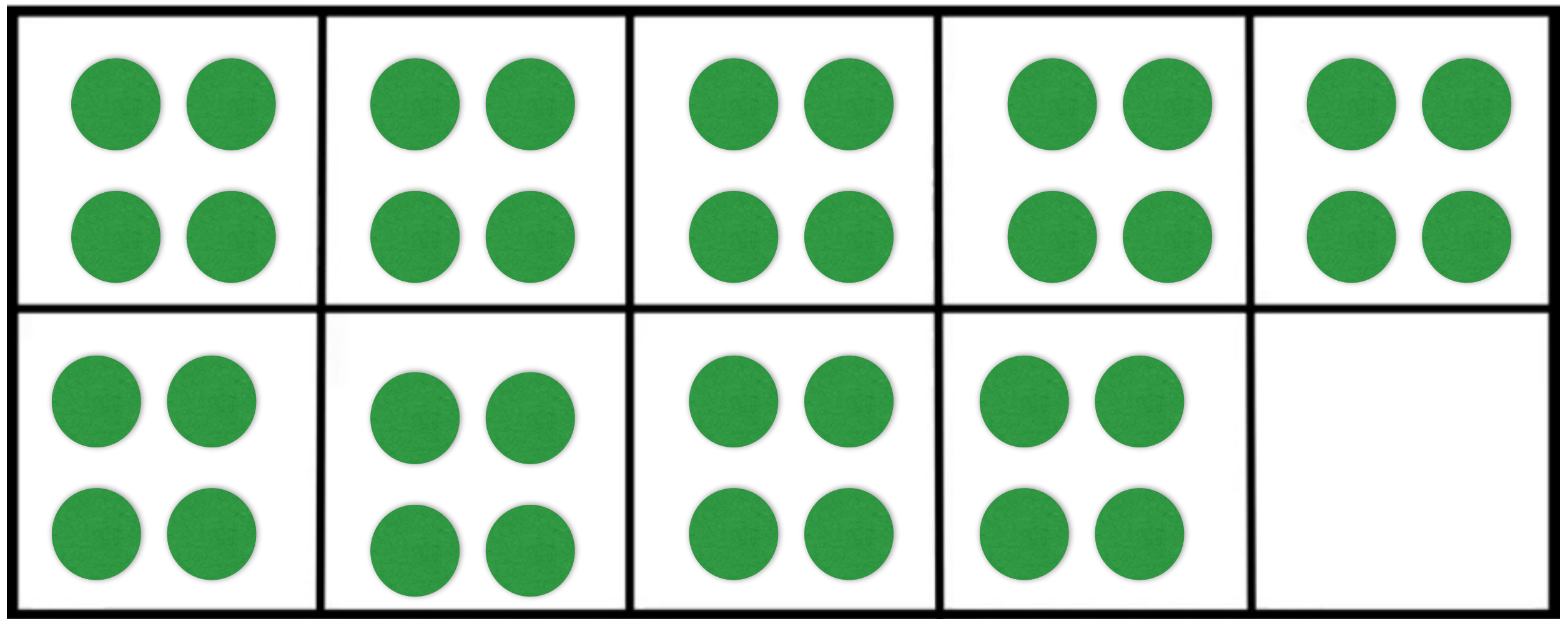
$$(8 \times 10) - 8 = 72$$





How many?

How did you see them?



How does thinking about the first image we saw help us with this new image?

Number Talks

Learning Intentions:

- develop multiple strategies for Decomposing (Mental Math)
- developing flexibility through use of multiple strategies
- Computational Fluency
- Place Value

10 - 15 minutes focussed on one question or a
“string” of questions

$$189 + 716 =$$

	6 x 600
	10 x 600
	16 x 600
	16 x 599

Number Talks Using Equations

Multiplication String

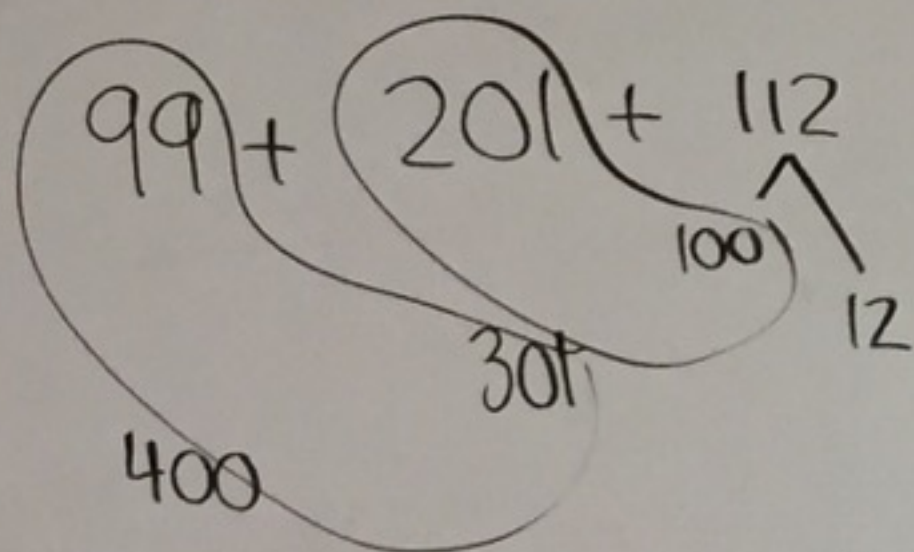
$$7 \times 7 =$$

<https://mathsolutions.wistia.com/medias/3flcbu6fnw>

3, 2017

$$300 + 112 = 412$$

$$99 + 112 + 201 = 402 \quad (412)$$



$$400 + 12 = 412$$

100

$$200 + 100 = 300$$

$$300 + 112 = 412$$

NUMBER TALKS

WHOLE NUMBER COMPUTATION



- More than 850 purposefully designed number talks
- Streaming video featuring 19 number talks filmed in actual classrooms



SHERRY PARRISH

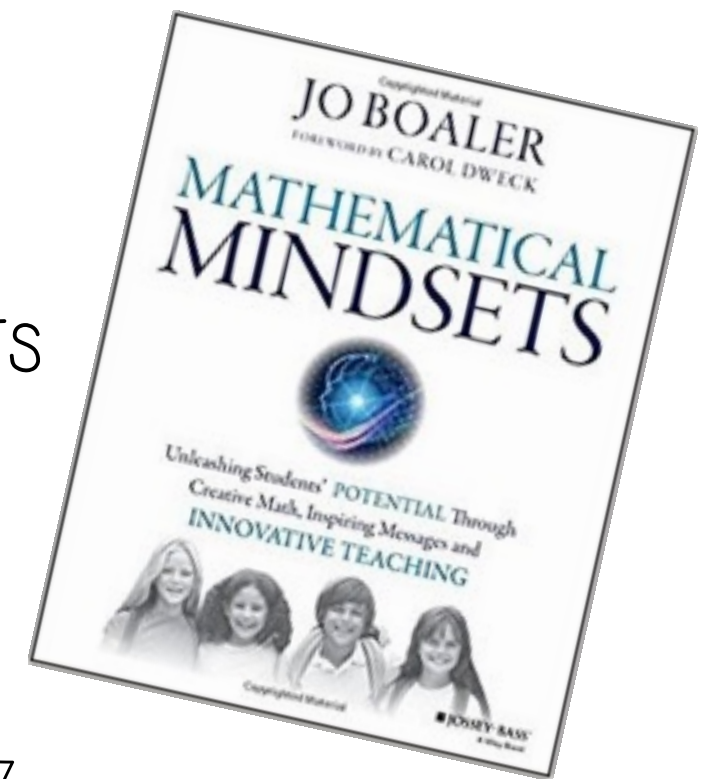
A Multimedia Professional Learning Resource



INCLUDES
VIDEO STREAMING

“Teachers are the most important resource for students. They are the ones who can create exciting mathematics environments, give students the positive messages they need, and take any math task and make it one that piques student curiosity and interest.”

– Jo Boaler, Mathematical Mindsets (2016), pg. 57



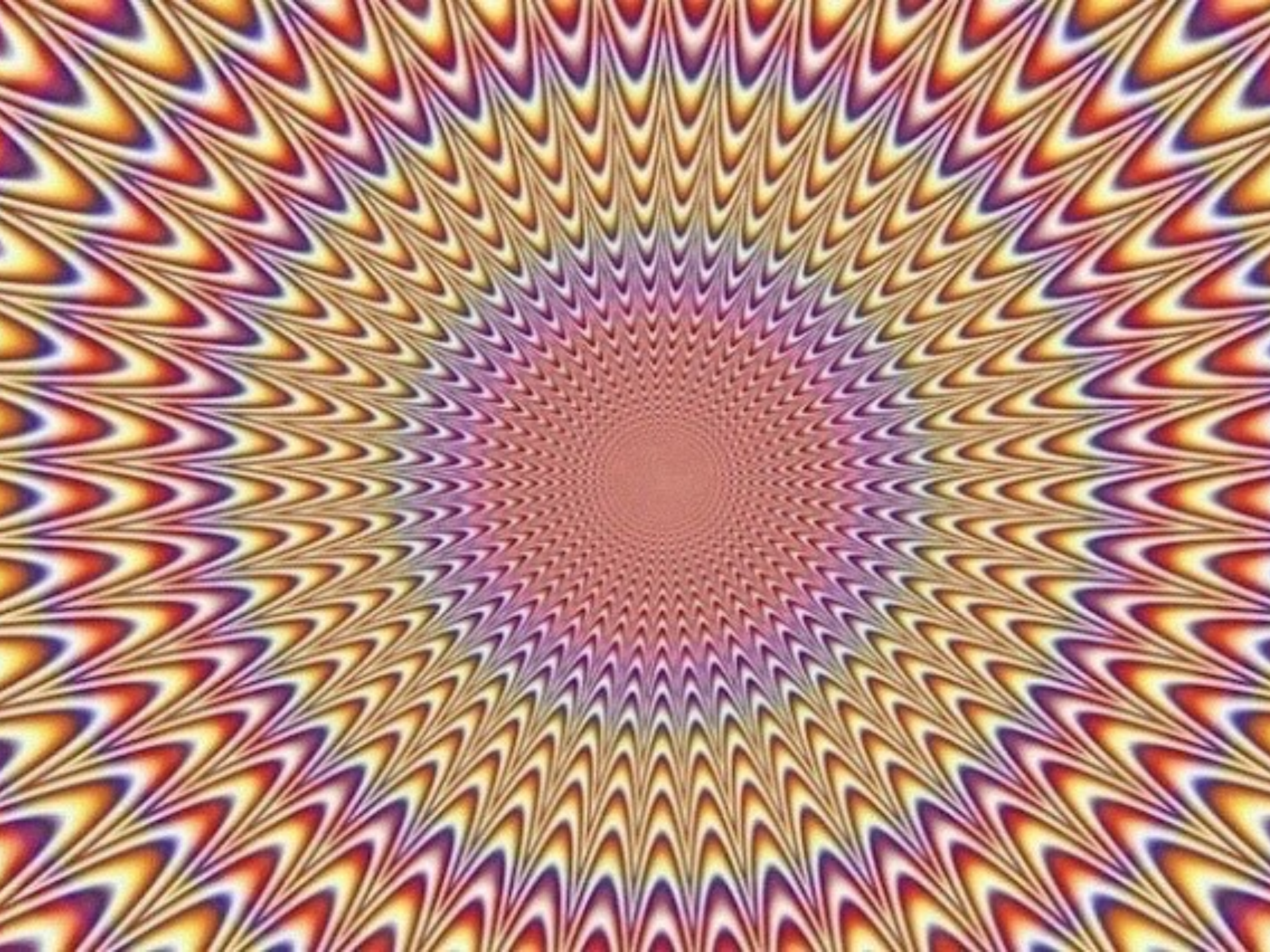
How do we design mathematical learning opportunities where all students feel successful and develop self-efficacy?

Finding Out What Students Know

Why is this important?

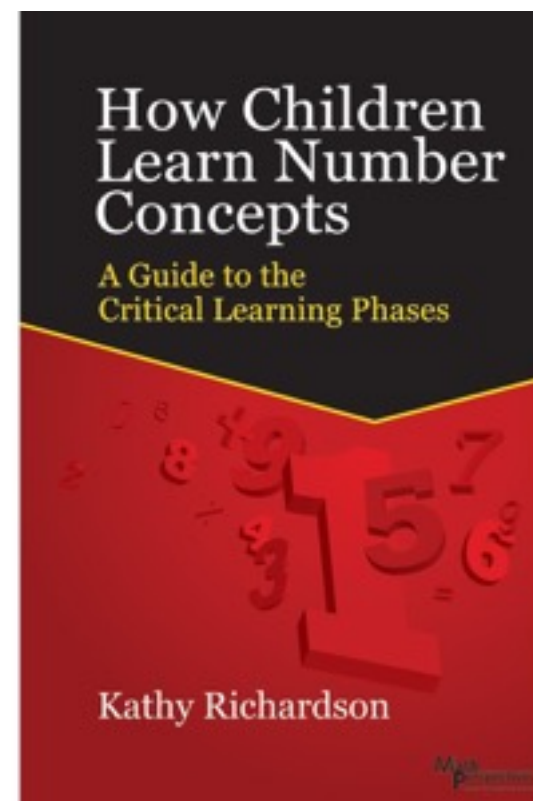
“When children are taught mathematical concepts and procedures before they reach certain levels of thinking, they do not see the underlying logic of mathematics. All they can do is memorize processes and procedures. It may appear that they know the mathematics, but in reality, this is just an illusion.”

~ Kathy Richardson (2012), How Children Learn Number Concepts, pg. xii



Learning is developmental...

Understanding Place Value: Numbers as Hundreds, Tens, and Ones (Extensions to Thousands)									
Adding and Subtracting Three-Digit Numbers									
Name									
1.		Adds and subtracts multiples of hundreds without counting							
2.		Knows parts of 1 hundred (in tens) without counting (Ex: 60 and 40, 70 and 30)							
3.		Mentally breaks apart 1 hundred into tens and ones, and recognizes that one of the tens will be broken up (Ex: 64 and 36)							
4.		When adding numbers that result in a hundred and some more, determines what part of the number is needed to make the next hundred and what part will be left over; combines the resulting hundreds, tens, and ones without counting							
5.		Breaks apart and recombines hundreds, tens, and ones when adding and subtracting							
6.		Comments							
7.									
8.									
9.									



Developmental progression of Multiplication

Recognizing Equal Groups

- Counts by equal groups (e.g., 2's, 5's, 10's, and so forth)
- Knows the quantity stays the same when counted by different sized groups (conservation of number)
- Identifies and extends the number pattern that emerge when counting by equal groups

Multiplying Equal Groups

- Counts groups as single entities (unitizes)
- Distinguishes between number of groups and number of objects in a group
- Show with models "a number of groups of a certain size" when the language of "groups of" is presented with various terms (e.g., "piles of," "stacks of," "rows of," "cups of,")
- Interprets word problems using models and drawings showing the number of groups and the number in each group
- Records number of groups in each step of a skip counting pattern, relating it to multiplication

Multiplying Using Rectangular Arrays

- Builds rectangular arrays using "rows of"
- Describes arrays in terms of equal groups (usually by rows)
- Partitions arrays into smaller arrays
- Describes arrays in terms of equal groups when the array is only partially visible

Using Multiplicative Thinking to Solve Single and Multi-Digit Multiplication Problems

- Repeated Addition
- Skip Counting
- Groups Of
- Array Models
- Uses multiplication to solve a problem for at least part of the answer and identifies the number of groups (for that part) when explaining the solution including:

- **Using known facts and compensating** e.g., $7 \times 8 =$

But I know $7 \times 7 = 49$

$$49 + 7 = 56$$

- **Doubling and Halving** e.g., for $50 \times 12 =$

$$100 \times 6 = 600$$

- **Making landmark or friendly numbers** e.g., for 9×7

$$+ 1$$

$$10 \times 7 = 70$$

$$70 - 7 = 63$$

- **Using the commutative property** (e.g., $3 \times 4 = 4 \times 3$)

- **Using the distributive property/partial products** e.g. 12×15

$$12 \times (10 + 5) =$$

$$12 \times 10 = 120$$

$$12 \times 5 = 60$$

$$120 + 60 = 180$$

*NOTE: The array model works really well to provide a visual for this strategy.

- **Breaking Factors into Smaller Parts/Associative Property** e.g., 12×25

$$(4 \times 25) + (4 \times 25) + (4 \times 25)$$

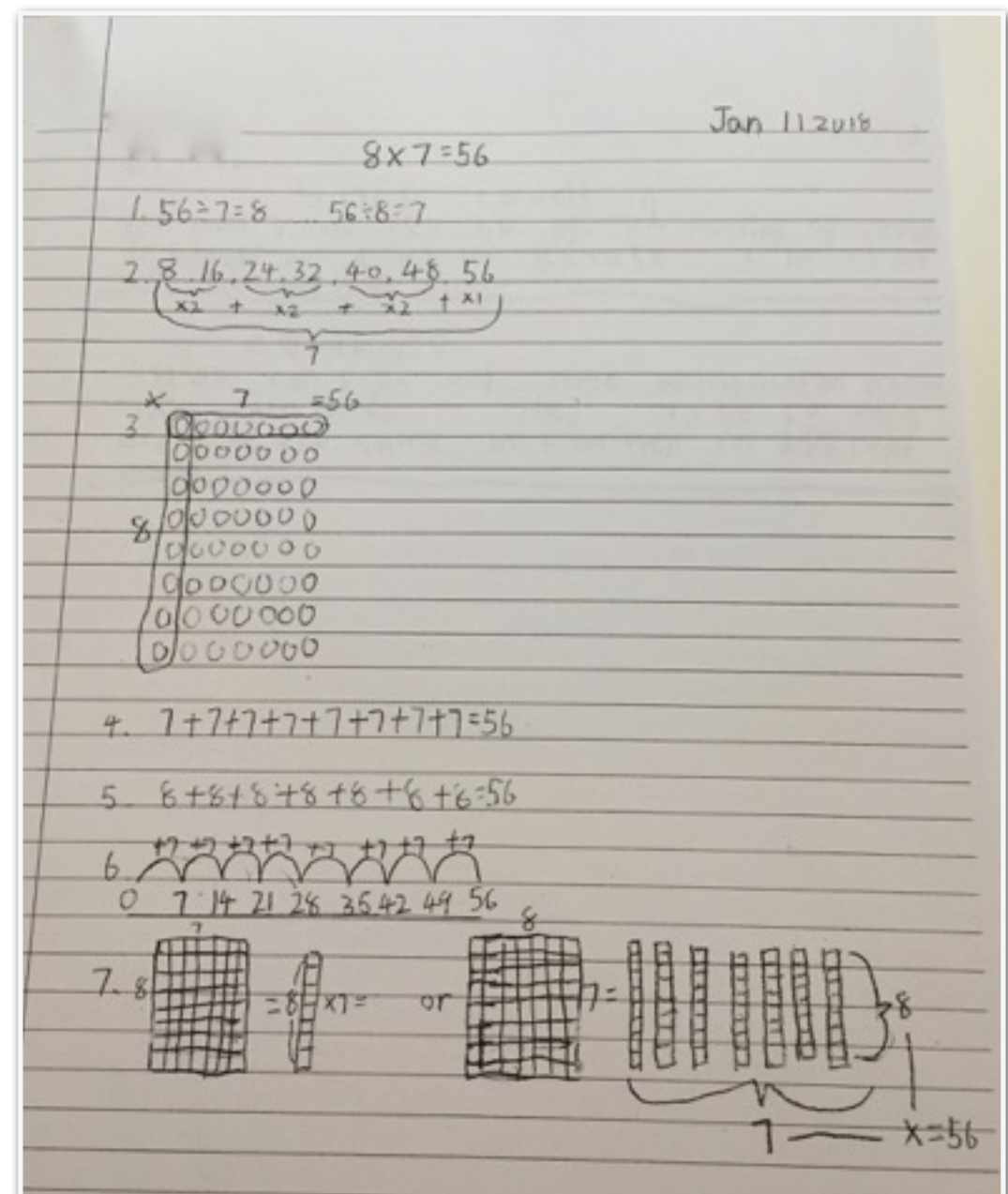
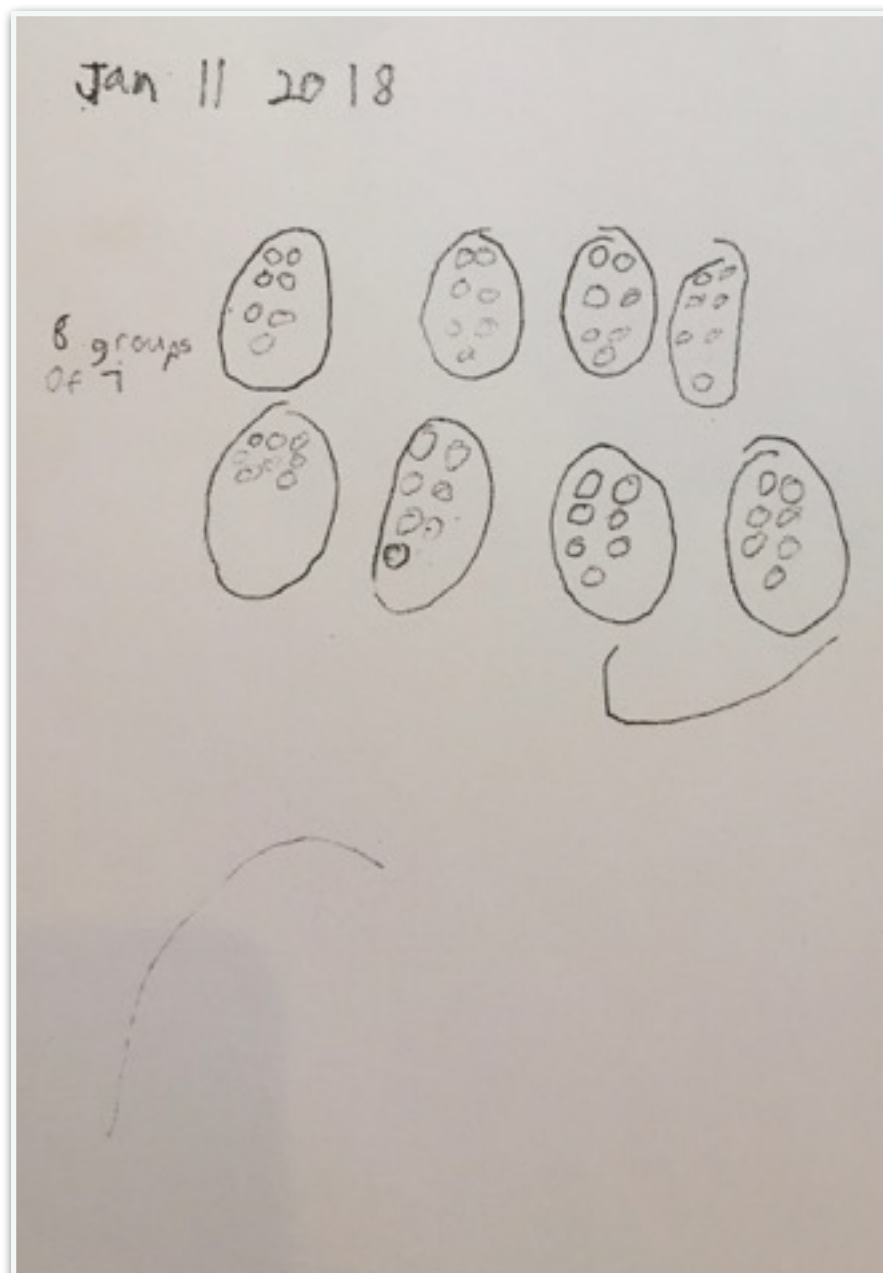
Help students to connect this to $3 \times (4 \times 25)$

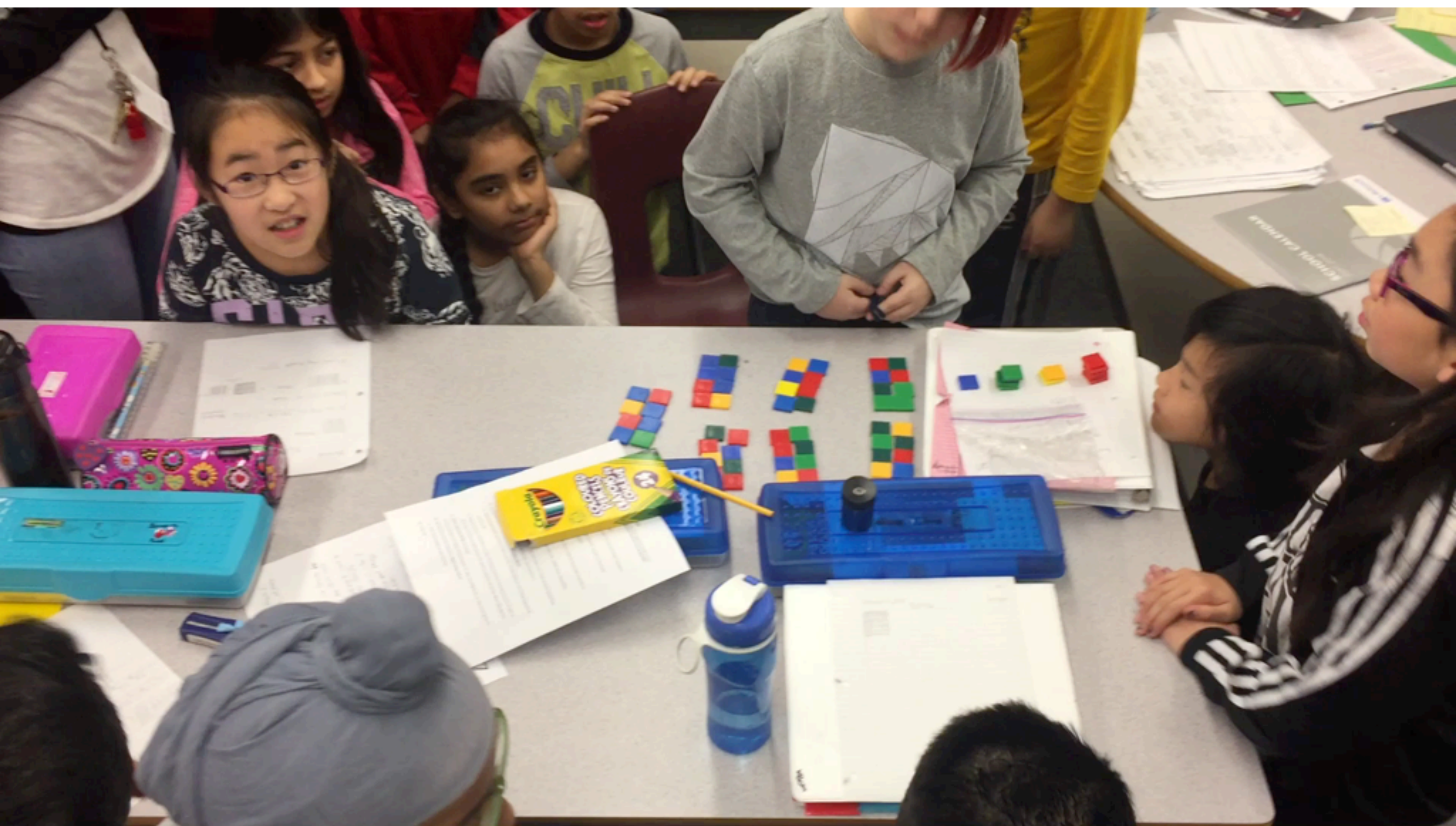
$$3 \times 100$$

Finding Out What Students Know

Activating Prior Knowledge

What are all the ways you can show $8 \times 7 =$

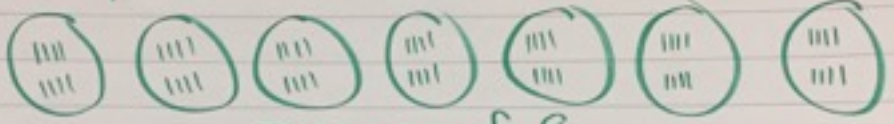




AFTER: Students share aloud the strategies they know!

Strategies for Multiplication

Groups of Model or Picture



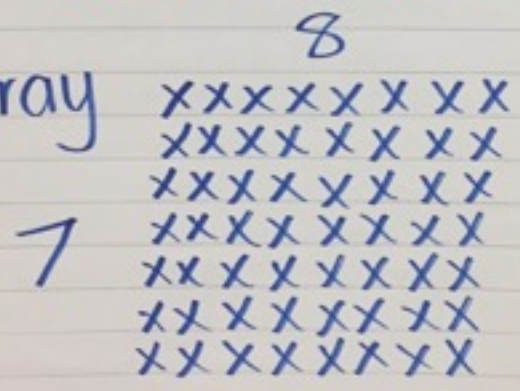
7 groups of 8

Skip Counting
8, 16, 24, 32, 40, 48, 56

Array

8

7



Related Facts

$$56 \div 7 = 8$$
$$56 \div 8 = 7$$

Using Facts I know / Friendly Facts

$$8 \times 7 = 56$$
$$8 \times 8 = 64$$
$$64 - 8 = 56$$

half of 8 is $\rightarrow 4 \times 7 = 28$ and $28 + 28 = 56$

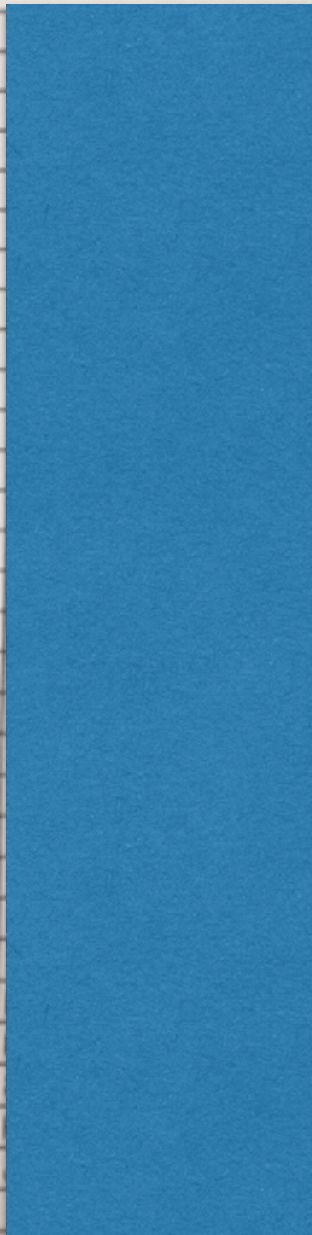
half of 8 is 4 half of 4 is 2 so $8 = 2 \times 2 \times 2$

$$56 \div 2 = 28$$
$$28 \div 2 = 14$$
$$14 \div 2 = 7$$

Commutative Principle

$$8 \times 7 = 56$$
$$7 \times 8 = 56$$

↑ ↑
Factors can be rearranged and the product is the same.



Groups of / Pictorial Rep	Repeated Addition	Skip Counting	Arrays	Commutative Property	Connection to Division	Friendly Facts	Connections to real world
		✓			✓	✓	
✓		✓					
✓							
	✓		✓	✓	✓	✓	
			✓				
✓	*	✓	✓		✓	✓	
		✓	✓		✓	✓	
✓							
✓	✓				✓	✓	
✓		✓		✓		✓	
	✓	✓	✓	✓	✓		
		✓	✓	✓	✓	✓	
			*	✓			
✓	✓		✓	✓	✓	✓	
	✓	✓	✓	✓	✓		
	✓		✓		✓	✓	
	✓			✓	*		
✓	✓	✓	✓	✓			
		✓	✓				
✓	✓	✓	✓	✓	✓	✓	
✓			✓			✓	
	✓			✓		✓	
✓				✓			
✓	✓		✓	✓	✓	✓	
✓	✓	✓	✓	✓	✓	✓	
	✓	✓	✓	✓	✓		
2	15	12	19	14	14	14	0

Lucas: "I know what times means but I can't explain it. I'm not good at math."

Me: Can you think of it as groups of?

Lucas: maybe

Me: Would drawing a picture help?

Did some arrays that weren't arrays

wrote $8 + 56 = 7$ and $7 \div 56 = 8$

Made a concrete model w/ tiles

Lucas: "I know what times means but I can't explain it. I'm not good at math."

Me: Can you think of it as groups of?

Lucas: Maybe

Me: Would drawing a picture help?

How might you plan your Multiplication unit?

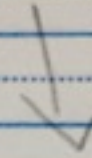


Talking Points

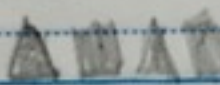
What are the critical understandings?

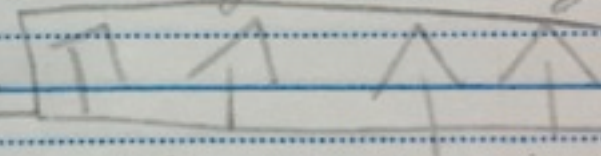
Talking Points	Rounds		
	Agree	Disagree	Unsure
Patterns are predictable.	A	A	
Patterns can increase (grow) but cannot decrease (shrink).	U	D	
Patterns can be made out of one item that is all the same colour (brown toothpicks).	D	A	
Numbers can be used to describe patterns.	U	U	
Patterns can help us to solve problems.	A	A	

1. Which talking point are you sure you were right with your answer?



I am pretty sure I'm right about patterns are predictable because if you draw two shapes you could just copy it so it is predictable

Example =  ← triangle and a square if you keep on adding to it, you will get the hang of it so it will be predictable.



2. Which talking point are you unsure about?

I am most unsure about "Numbers can be used to describe patterns" because I don't remember learning that last year.

② Patterns can increase
but cannot decrease.
I do know that
Patterns can increase
but before I
never knew that Patter-
ns can decrease.

① I was sure about
that Patterns can
be made out of
Items that are all
the same color
because you can
make different design
that can be used
in different ways.

September 26 2017

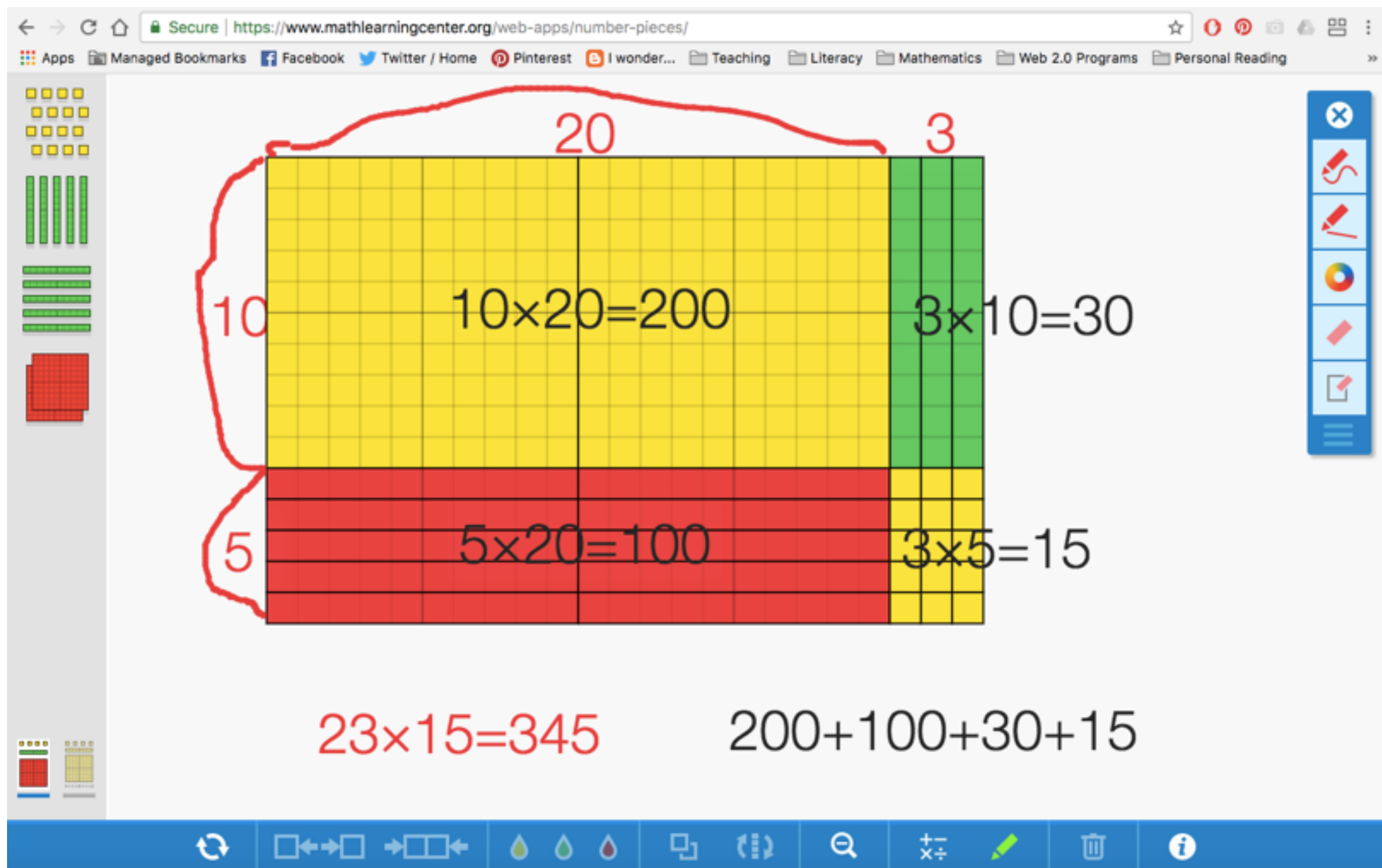
1. The talking point I am sure I am right with my answer is Patterns are predictable because you can always predict what comes next when you do 3 or 2 shapes.
2. The talking point I am unsure about is Numbers can be used to describe patterns because I really don't get it and I don't know if it is a disagree or a Agree so I just wrote unsure/U.

① The talking point I am sure is right is patterns can be made out of one item that is all the same colour brown to other people.

② Which talking point are you unsure about that is patterns can help us to solve problems.

Introducing New Concepts

Conceptual Understanding of 2-digit by 2-digit multiplication using www.mathlearningcenter.org to show Partial Products



$$2.38 \rightarrow \begin{array}{c} \boxed{\begin{array}{cc} \text{[10x10 grid]} & \text{[10x10 grid]} \end{array}} \\ 2 \end{array} + \begin{array}{c} \boxed{\begin{array}{c} \text{[1x10 grid]} \\ \text{[1x10 grid]} \\ \text{[1x10 grid]} \end{array}} \\ .3 \end{array} + \begin{array}{c} \boxed{\begin{array}{c} \text{[1x10 grid]} \\ \text{[1x10 grid]} \end{array}} \\ .08 \end{array}$$

$$2.56 \rightarrow \begin{array}{c} \boxed{\begin{array}{cc} \text{[10x10 grid]} & \text{[10x10 grid]} \end{array}} \\ 2 \end{array} + \begin{array}{c} \boxed{\begin{array}{c} \text{[1x10 grid]} \quad \text{[1x10 grid]} \quad \text{[1x10 grid]} \\ \text{[1x10 grid]} \quad \text{[1x10 grid]} \end{array}} \\ .5 \end{array} + \begin{array}{c} \boxed{\begin{array}{c} \text{[1x10 grid]} \\ \text{[1x10 grid]} \end{array}} \\ .06 \end{array}$$

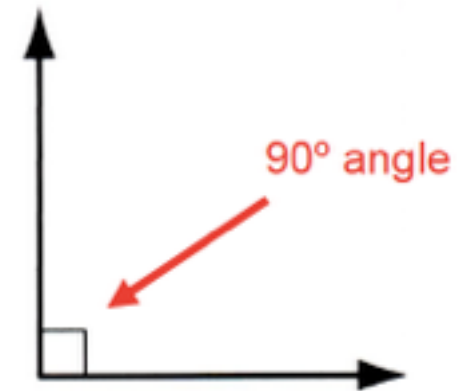
Introducing Vocabulary

Math Word Walls

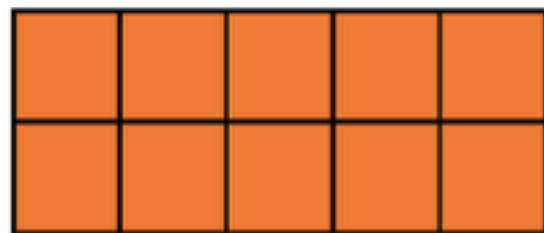
polygons



right angle

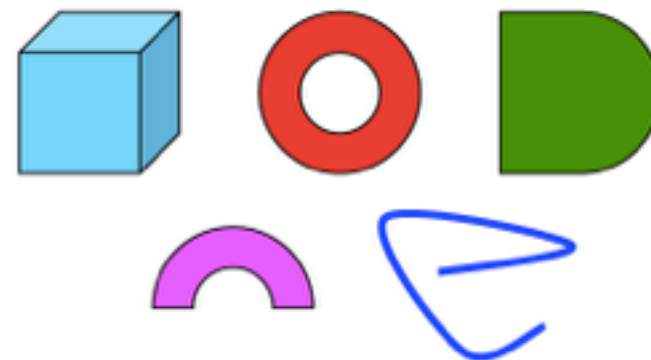


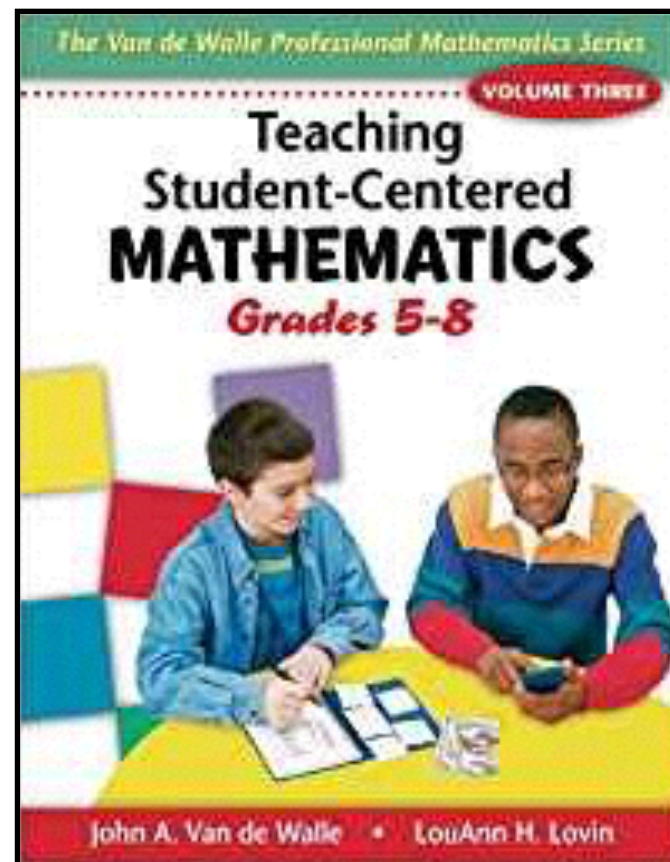
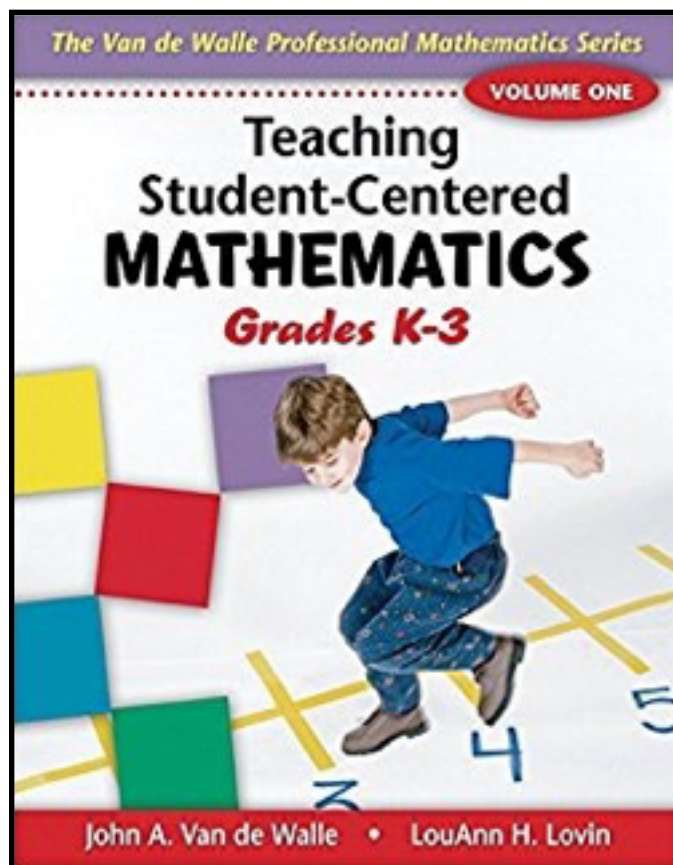
perimeter



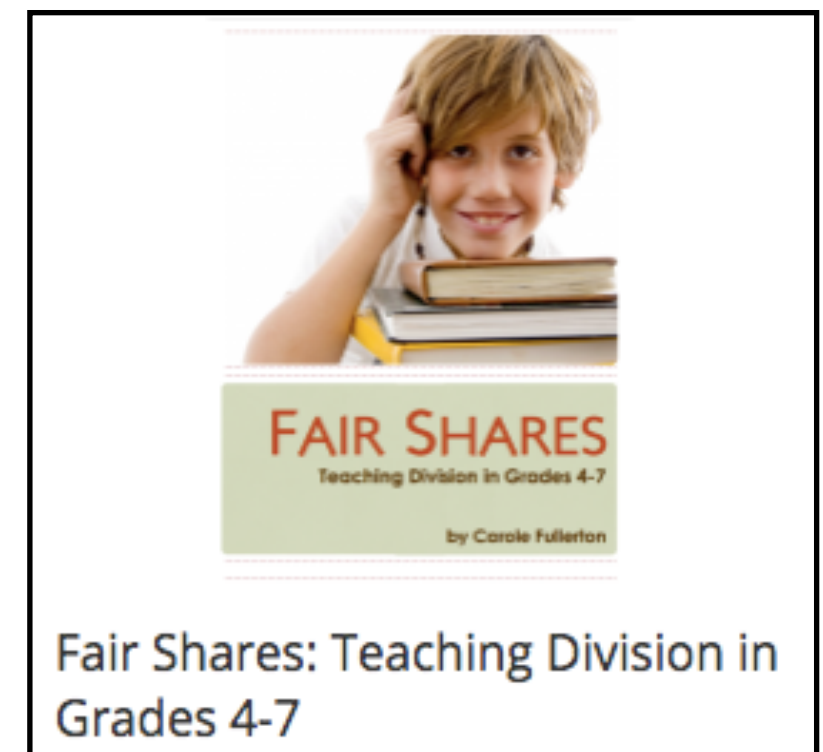
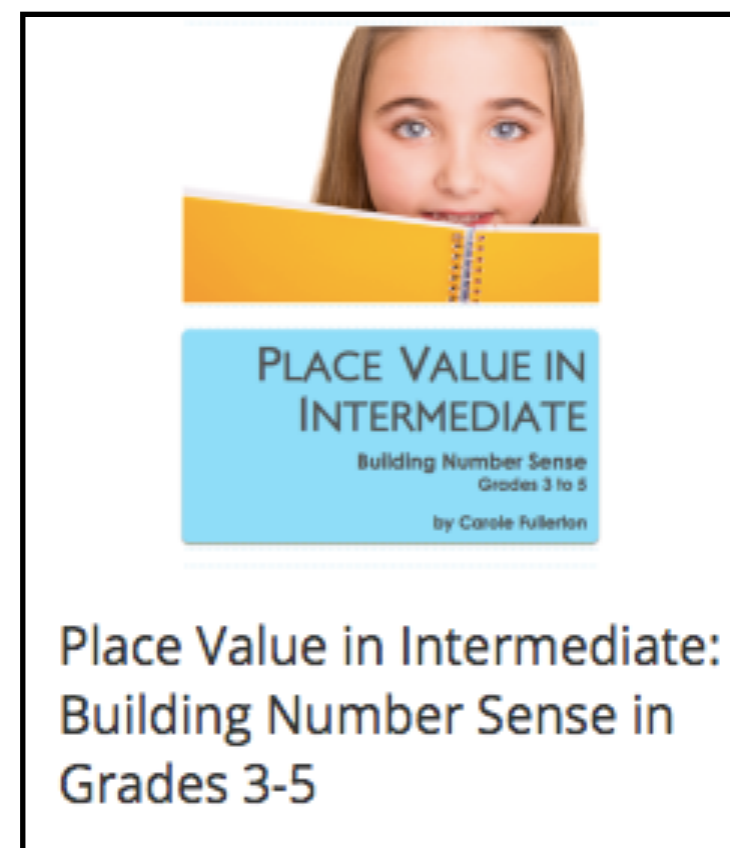
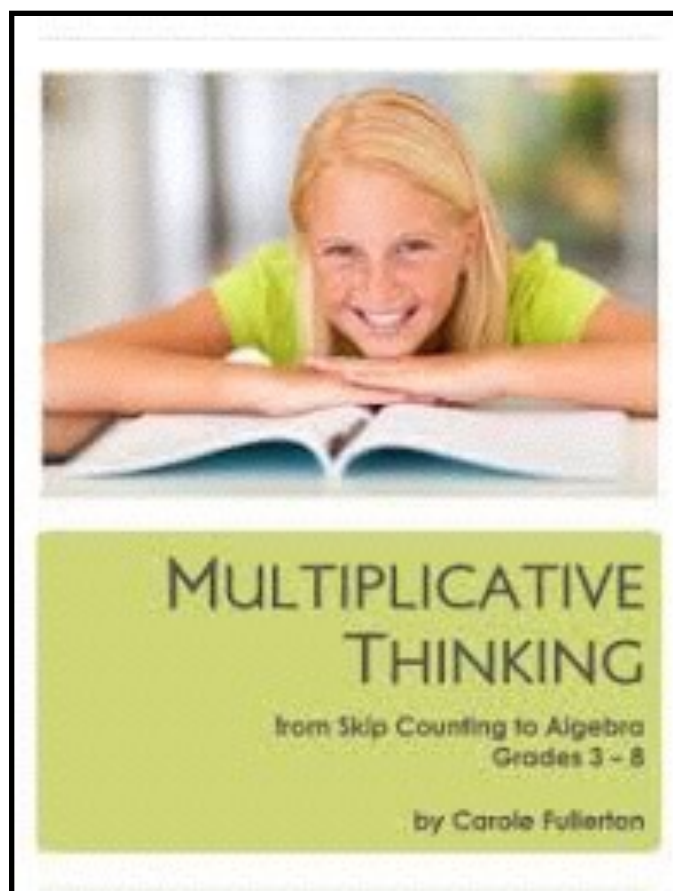
$$P = 5 + 2 + 5 + 2 = 14 \text{ cm}$$

non-polygons

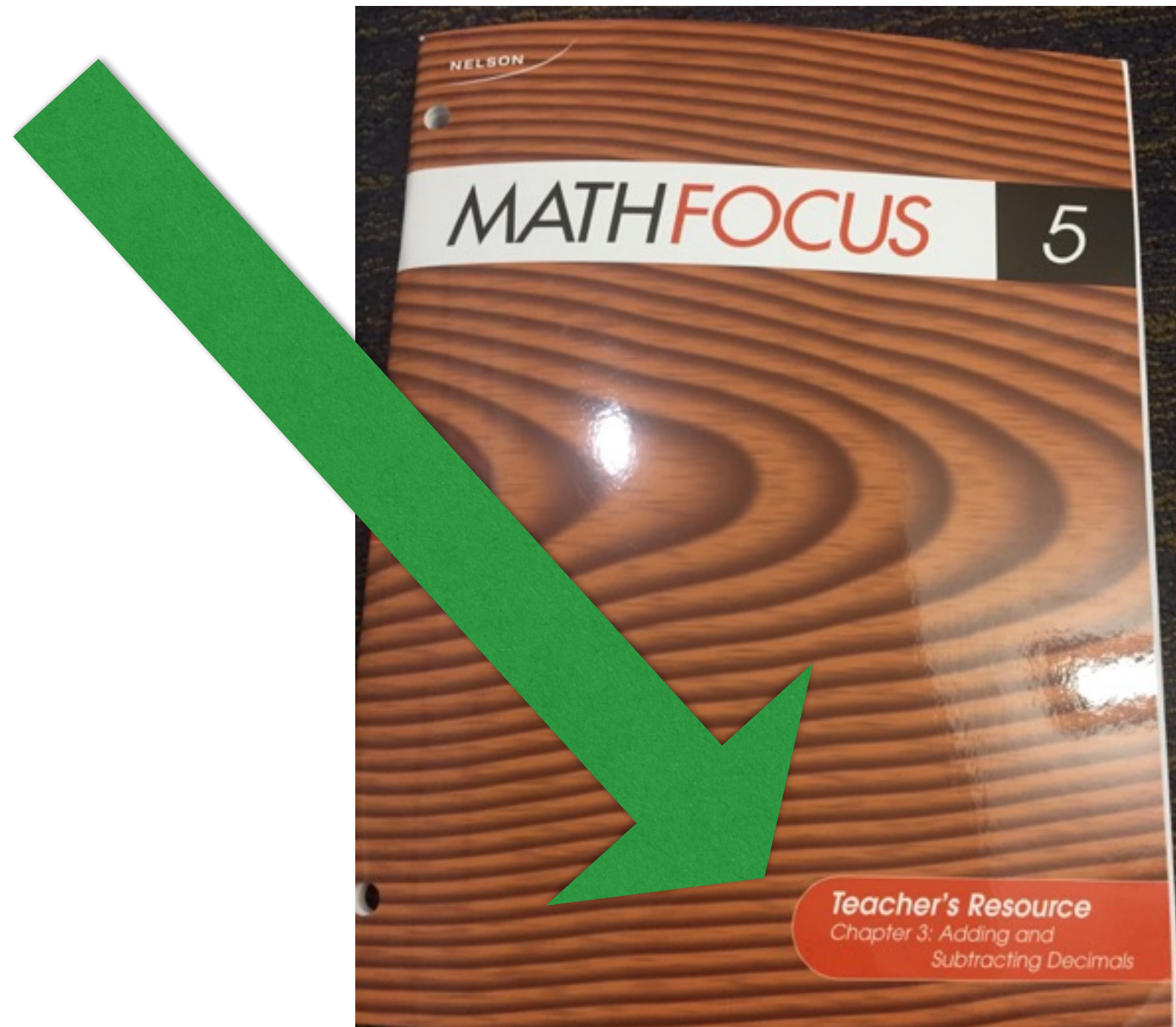




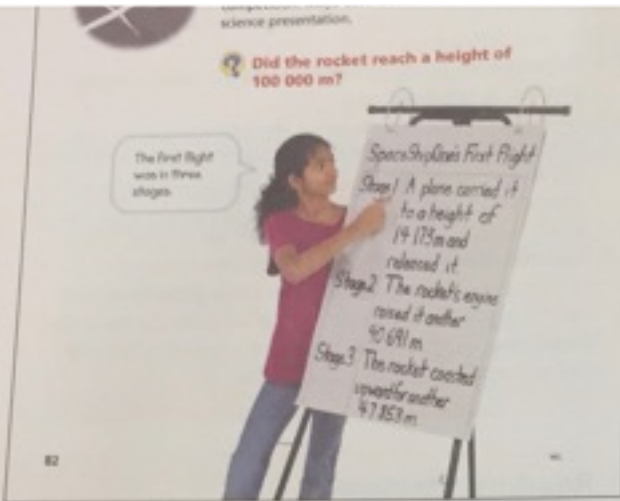
These are key resources that will help teachers develop an understanding how to teach in conceptual ways.



Textbook Whole Class



Three Part Lesson Format



science presentation.

Did the rocket reach a height of 100 000 m?

The first flight was in three stages.

SpaceShyline's First Flight

Stage 1: A plane carried it to a height of 19 173 m and released it.

Stage 2: The rocket's engines raised it another 40 691 m.

Stage 3: The rocket coasted upward for another 47 853 m.

Building Strategies

Visualizing

What distances can you estimate or measure to help you visualize how far 100 000 m is?

A. How might Maya use her number line to estimate the height of the rocket at the end of Stage 2?

B. At the end of Stage 1, about how much higher did the rocket have to go to reach 100 000 m? Explain your method.

C. At the end of Stage 2, about how much higher did the rocket have to go to reach 100 000 m? Explain your method.

D. Did the rocket reach a height greater than 100 000 m by the end of Stage 3?

Reflecting

E. How do you think Maya knew her estimate would be less than the rocket's actual height at the end of Stage 2?

F. Did you need to calculate an exact answer to figure out if the rocket reached a height of 100 000 m? Explain.

1 Introduction
(Whole Class/Small Group)
♦ 5–10 min

Write the calculation $126 + 385$ on an overhead, on the board, or on an interactive whiteboard. Ask students to use base ten blocks to model each number.

Sample Discourse

"How can you estimate $126 + 385$ using just the front digits?"

- $100 + 300 = 400$

"How can you estimate $126 + 385$ using the hundreds and the tens?"

- $100 + 300 = 400$, $20 + 80 = 100$, so $126 + 385$ is about $400 + 100$ or about 500

"What other ways can you estimate $126 + 385$?"

- I can use nearby numbers to estimate: $125 + 375 = 500$.
- I can use rounding to hundreds estimate: $100 + 400 = 500$.

2 Teaching and Learning
(Whole Class/Pairs) ♦ 25–35 min

Explain to students the significance of a reusable rocket that is able to travel 100 000 m into the atmosphere. The approximate distance a rocket would need to travel in order to leave Earth's atmosphere and enter suborbital (outer) space is 100 000 m. Because of the tremendous costs associated with space flight, scientists continue to look to private industry to produce efficient means of space travel. The reason each rocket needs to be launched twice is to prove that the rocket is reusable.

Together with students, read the information and the central question on Student Book page 82. Distribute number lines to each pair of students. Work through Maya's Solution together as students locate the estimated numbers and show the addition on their own number lines. Note that Maya could have rounded 40 691 to 41 000 to estimate. However, rounding down was a good idea in this situation because she'd be sure that if her estimate was over 100 000, then the calculation would be over 100 000. Students can use different coloured pencil crayons to represent different parts of the estimation process. Explain that Maya rounded each of the heights, and then added the rounded numbers to determine an estimate. Students can complete Prompts A to D in pairs.

Answers to Reflecting Questions

- E. For example, she rounded 14 173 down to 14 000 and 40 691 down to 40 000, so she knows that her estimate of $14\ 000\text{ m} + 40\ 000\text{ m}$ is less than the actual height of $14\ 173\text{ m} + 40\ 691\text{ m}$.
- F. No. For example, I just had to estimate if the total height was at least 100 000 m. My estimates showed that the rocket travelled higher than 100 000 m, so I didn't have to calculate an exact answer.

3 Consolidation 10–15 min

Checking (Pairs)

Provide additional number lines as needed. Students should be able to solve this problem by rounding the numbers appropriately and using number lines to add the rounded heights. The number line will allow students to **visualize** their estimates.

Practising (Individual)

3. If extra support is required, guide those students and provide copies of **Scaffolding p. 71**.
5. & 6. Guide students to understand that they are estimating differences in these two questions.
8. Students should understand that 7185 is closer to 7000 than to 8000, so it makes sense to round 7185 down to 7000. 2835 is closer to 3000 than to 2000, so it makes sense to round 2835 to 3000.
9. Students can estimate each number on the bar graph by using the number on the horizontal line closest to the height of each bar.

Answer to Key Question

6. For example, about 4500 m; Since 6526 m is close to 6500 m, $6500\text{ m} + 3500\text{ m} = 10\ 000\text{ m}$, and 924 m is about 1000 m, the vehicle would have to dive about $3500 + 1000 = 4500\text{ m}$ more to reach the bottom.

Closing (Whole Class)

Question 10 allows students to reflect on and consolidate their learning for this lesson. Have students share their estimates with the rest of the class. Discuss which estimates are overestimates and which are underestimates. Determine the exact amount of the sum, and compare estimates that were closest to the exact amount.

Answer to Closing Question

10. For example, to estimate $43\ 765 + 56\ 239$:

(1) I can use compatible numbers.

$$44\ 000 + 56\ 000 = 100\ 000; \text{ so}$$

$$43\ 765 + 56\ 239 \text{ is about } 100\ 000.$$

(2) I can round one number up and one down.

$$45\ 000 + 55\ 000 = 100\ 000; \text{ so}$$

$$43\ 765 + 56\ 239 \text{ is about } 100\ 000.$$

For example, to estimate $56\ 239 - 43\ 765$:

(1) I can use compatible numbers.

$$56\ 000 - 46\ 000 = 10\ 000$$

(2) I can round one number down and one up:

$$56\ 000 - 44\ 000 = 12\ 000$$

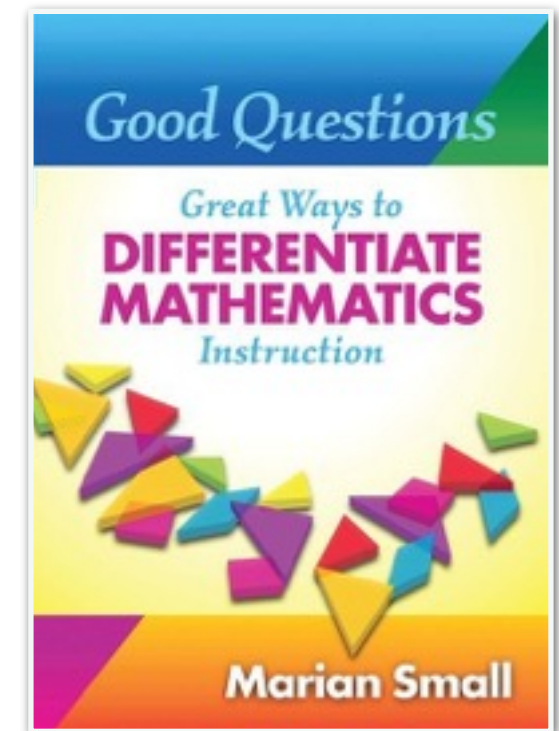
Follow-Up and Preparation for Next Class

Have students think about how they would communicate how to tie their shoelaces to someone who doesn't know how to tie them. What would they include in their explanation?

Open Questions

Why use this approach?

“The teacher can create a single question or task that is inclusive of not only allowing for different students to approach it using different strategies, but also in allowing for students at different stages of mathematical development to benefit and grow from attention to the task.”



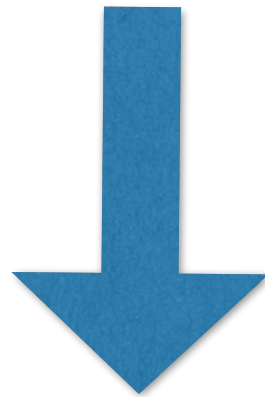
~ Marian Small (2012) Good Questions: Great Ways to Differentiate Mathematics Instruction

Open Questions

How do I create an open question?

Turning around a question

What is half of 100?



50 is a fraction of a number.
What could the fraction and number be?

Work backwards...

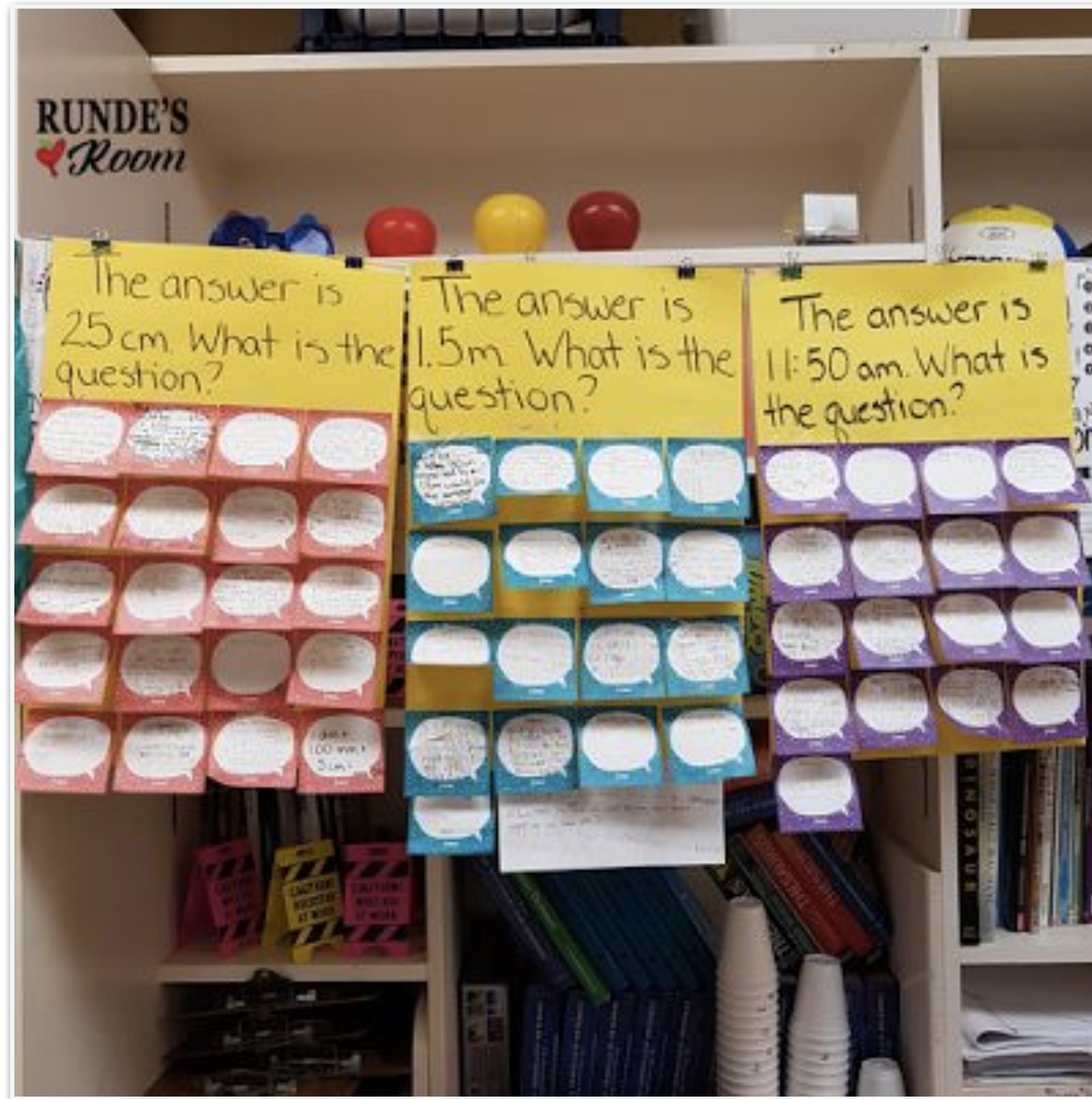
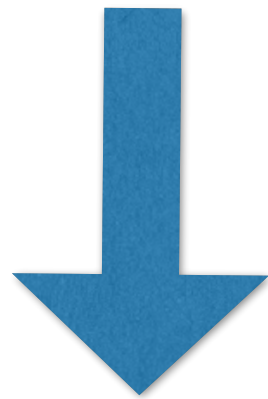


Image retrieved from: <http://www.rundesroom.com/2017/02/5-activities-for-teaching-problem.html>

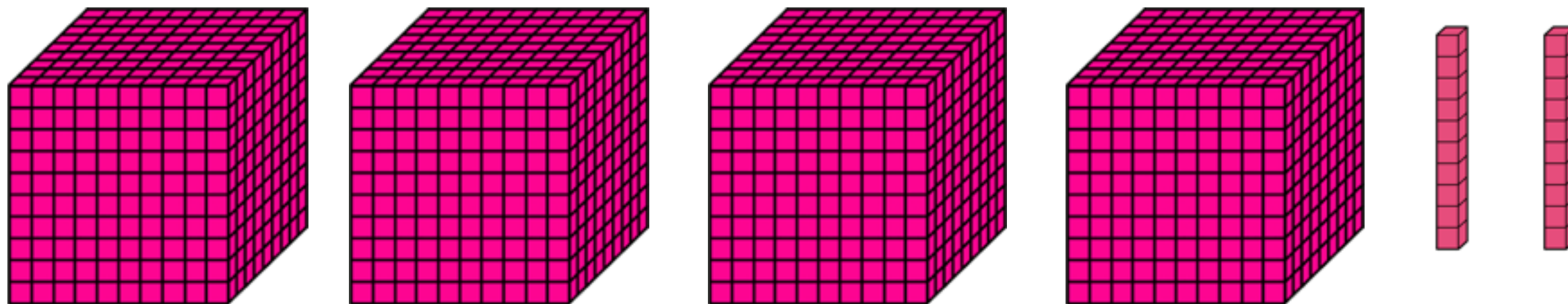
Change the Question

What number has 2 thousands, 3 hundreds, 2 tens, and 9 ones?



You can model a number with 6 base 10 blocks.
What could the number be?

4020



Or many other numbers!



Open Questions

What resources are available?



Each book spans several Grades (e.g., K - 3, 4 - 6, and 7 - 9).
Currently only the Number Strand is aligned to our BC Curriculum.
The other strands are coming in Spring 2018.

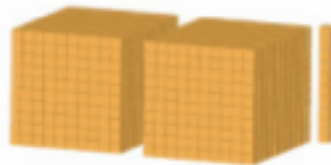
GETTING STARTED



Q You represent a number using base ten blocks with twice as many thousands blocks as tens blocks. What might the number be?

SAMPLE RESPONSE

2010 **OR** 8143 **OR** 6632



Tell as many mathematical ideas as you can about the number $5 \square 6$.

SAMPLE RESPONSE

It is greater than 500, it is less than 600, and it is even. **OR**
It is greater than 200 and less than 1000.



Create a four-digit number so that

- two digits are 7s
- two digits are 3s
- one of the 7s is 10 times as great as the other 7
- one of the 3s is 10 times as great as the other 3

SAMPLE RESPONSE

7733 **OR** 3377



You see the number 4102 on an Internet news site. What might that number describe?

SAMPLE RESPONSE

It might be an address. **OR**
It might be the number of people at an event at a small arena.



When you read a number, you say the word *thousand* but not the word *hundred*. For example, it could be *three thousand twenty-two*; it could not be *four hundred twenty*. List several numbers it could be and several numbers it could not be.

SAMPLE RESPONSE

It could be 3025, 4015, or 7023.
It could not be 3125, 4115, or 712.

WORKING ON IT

Q Write five or more four-digit numbers. Each number should have at least one 0. Write each number in expanded form. What do the expanded forms have in common?

SAMPLE RESPONSE

3032 is 3 thousands + 3 tens + 2 ones.
9902 is 9 thousands + 9 hundreds + 2 ones.
4089 is $9 + 80 + 4000$.
2100 is $2000 + 100$.
9230 is 9 thousands + 2 hundreds + 3 tens.
The expanded forms all have three parts.
The expanded forms all have a thousands portion.

Q Tell three or more things about the number 3□□2. The numbers in the blanks can be any digit from 0 to 9, and they can be the same or different digits.

SAMPLE RESPONSE

The number is at least 3002.
It is an even number.
The number is less than 4000.
The greatest the number could be is 3992.

Q Tell as many things as you can about the number 10 000.

SAMPLE RESPONSE

It is 100 hundreds.
It is 1000 tens.
It is 10 thousands.
It is more than 9999.
You say it when you skip count by 10s starting at 0.
You say it when you skip count by 25s from 0.

CONSOLIDATING



Q When and why might it be useful to use expanded notation to describe a number? For example, 1204 as $1000 + 200 + 4$ or 1 thousand + 2 hundreds + 4 ones.

SAMPLE RESPONSE

You can use it to explain how you add four-digit numbers and why you add the thousands together, the hundreds together, the tens together, and the ones together.

Q Do you think there are more ways to represent the number 3005 or the number 3006? Why do you think that?

SAMPLE RESPONSE

I think there are many ways to represent both numbers. I doubt that there are more ways to represent one number than the other. I think that no matter how I represent 3005, I could add 1 to show 3006, so I think the number of representations is about the same. **OR**

I think there are more ways to represent 3006. You can use $1 + 3005$, $2 + 3004$, $3 + 3003$, $4 + 3002$, ... $3005 + 1$. Using a pattern like that, you can see that there is at least one more way to represent 3006 than 3005.

Q What do you know for sure about a number if you say the word *thousand* but not the word *hundred* when you read it?

SAMPLE RESPONSE

I know there is not a 0 in the thousands place if the number is less than 10 000, but there is a 0 in the hundreds place.

Open Questions

<http://www.onetwainfinity.ca/presentations/AMElemNov.pdf>

Think about that child in your class that seems to struggle the most.

How would he/she participate in *this* activity?

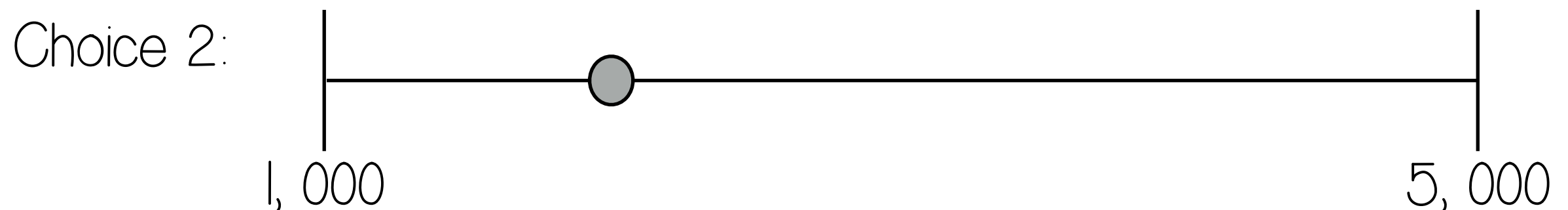
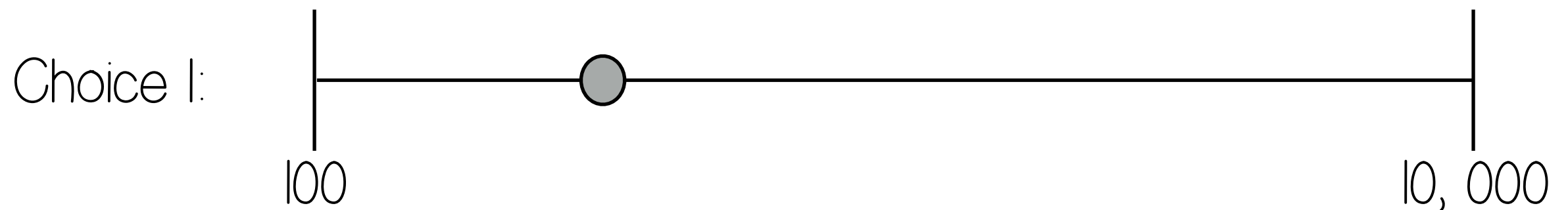


Can everyone “get in” at their developmental stage?

Parallel Tasks

Parallel tasks are a set of two or three tasks that are designed to meet the needs of students at different developmental levels, but that get at the same big idea and are close enough in context that they can be discussed simultaneously.

What value do you think the dot on the number line represent?



Open Middle Problems

These questions have a 'closed' beginning and a 'closed' ending but how students approach the question can occur in different ways.

Open Middle

<http://www.openmiddle.com/>

The area of a rectangle is 24 square units. What is the smallest perimeter the rectangle could have?

How might you start?



Open Middle

Challenging math problems worth solving

[Home](#)[Kinder ▾](#)[Grade 1 ▾](#)[Grade 2 ▾](#)[Grade 3 ▾](#)[Grade 4 ▾](#)[Grade 5 ▾](#)[Grade 6 ▾](#)[Grade 7 ▾](#)[Grade 8 ▾](#)[High School ▾](#)[About](#)[Submit](#)[Operations & Algebraic Thinking](#)[Number & Operations in Base Ten](#)[Number & Operations—Fractions](#)[Measurement & Data](#)[Geometry](#)

THE TOP 10 MOST VIEWED PROBLEMS OF 2016

1. [Two-Step Equations](#) by Audrey Mendivil, Daniel Lueva
2. [Order of Operations](#) by Robert Kaplinsky with answer
3. [Dot Card Counting](#) by Dan Meyer
4. [Rational and Irrational Numbers](#) by Bryan Anderson
5. [One Solution, No Solutions, Infinite Solutions](#) by Bryan Anderson
6. [Multiplying a Two-Digit Number by a Single-Digit Number](#) by Robert Kaplinsky
7. [Exponents and Order of Operations](#) by Zack Miller
8. [Converting Between Fractions and Decimals](#) by Robert Kaplinsky
9. [Interpreting Percentages](#) by Robert Kaplinsky
10. [Two-Step Equations 3](#) by Erick Lee



OPEN MIDDLE WORKSHEET

[Download the Open Middle Worksheet \(Regular\): Version 1.2](#)

[Download the Open Middle Worksheet \(Large\): Version 1.1](#)

BROWSE BY DEPTH OF KNOWLEDGE LEVEL

[DOK 2: Skills and Concepts](#)

[DOK 3: Strategic Thinking](#)

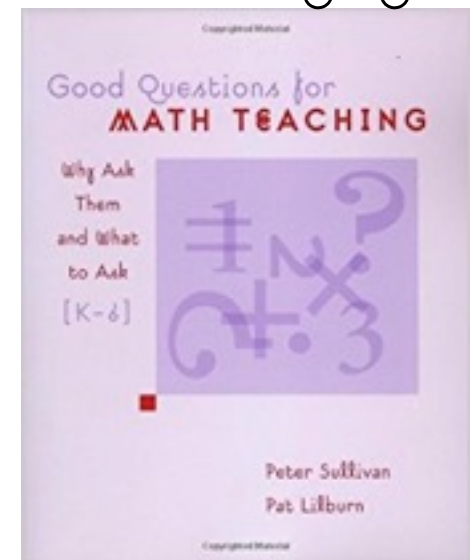
BROWSE BY COMMON CORE STATE STANDARDS

[Kindergarten \(10\)](#)

Rich Tasks

Rich tasks enable students to work mathematically by allowing them to:

- Step into activities even when the route to a solution is initially unclear
- Get started and explore because the tasks are accessible to pupils of wide ranging abilities
- Pose as well as solve problems, make conjectures
- Work at a range of levels
- Extend knowledge or apply knowledge in new contexts
- Work successfully when using different methods
- Broaden their problem-solving skills
- Deepen and broaden mathematical content knowledge
- See and make sense of underlying principles or make connections between different areas of mathematics
- Work within include intriguing contexts
- Observe other people being mathematical or see the role of mathematics within cultural settings



~NRICH Mathematics, Steve Hewson (2011)

My Favourite Places to go for Rich Math Tasks

Created by Margie Pearse

My Fav Places to go to Find Rich Math Tasks created by Margie Pearse @pearse_margie ...

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	A	B	C	D	E	F	G	H
1		My Fav Places to Find Rich Math Tasks by Margie Pearse & Lane Walker						
2	Title and Link	Description	CCSS*	accepting** submissions	Author	Twitter Handle		
3	Agree or Disagree	Gr 6-12 images are designed to start robust discussion	Yes	Yes	Tim McCaffrey	@timsmccaffrey		
4	Andrew Stadel	Spreadsheet of his 3-acts, 180's and other activities			Andrew Stadel	@mr_stadel		
5	Balanced Assessment	K+ investigations that are engaging and open-ended	No	No	Harvard Graduate School of Education			
6	Breakout EDU	Immersive learning games platform for purchase	No	No				
7	https://clotheslinemath.com/	The Clothesline is a manipulatable number line that makes	No	Yes	Andrew Stadel			
8	http://www.conceptuamath.com/math-t	Interactive math tools, 3-Act Math Tasks, and rich problems	Yes	No	Conceptua Math			
9	Daily Desmos	Intriguing activities designed to build graphing skills	No	No		@desmos		
10	Dan Meyer 3-Acts	Spreadsheet of K-12 activities	Yes	No	Dan Meyer	@ddmeyer		
11	https://emergentmath.com/my-problem	Common Core Problem-Based Problems based on Grade Level	Yes	Yes				
12	EMMaths Puzzles	Puzzles grouped by category	No	No	Emma Bell	@El_Timbre		
13	Estimation 180	Gr 4-8 Building number sense through estimation	Yes	No	Andrew Stadel	@mr_stadel		
14	http://www.expeditionarymath.com/	Gr 3+ lessons, wonders, explorations to discover	No	No		@livethemath		
15	Fraction Talks	Simple visuals to foster creative thinking around fractions	No	Yes	Na Banting	@NatBanting		
16	GFletchy 3-Acts Lessons	K-8,G 3-acts	Yes	No	Graham Fletcher	@gfletchy		
17	Got It -> Get It Transfers	Gr 7-Algebra 2 Replace a direct instruction lesson with a simple activity	Yes	Yes	Lane Walker	@LaneWalker2		
18	Heinemaann Pblm of Week	Primary to Geometry - Open-ended interesting problems	Yes	No	The Math Forum	@themathforum		
19	https://hungryteacher.com/	Real World Math Activities that promote student engagement, collaboration, and problem solving	Yes	No	Zack Patterson	@misterpatterson		

https://docs.google.com/spreadsheets/d/lyGaZy9g8X0HHFuWMBQkFI4pVStu_SlBnbZSkxo9nWPI/edit#gid=0

Rich Tasks: This is a DRAFT version

Created by Deanna Lightbody for the Provincial Numeracy Project

Websites - Problems/Rich Tasks





Site	Link	Description
NRICH Math	https://nrich.maths.org	K-12 Problems, Articles and Games
Galileo	http://galileo.org/classroom-examples/math/math-fair-problems/	K-12 Problems for Math Fairs and classroom use
Open Middle	http://www.openmiddle.com/	K-12 Challenging math problems, multiple ways to approach and ultimately solve the problem.
Three-Act Math Tasks	https://tapintoteenminds.com/3act-math/	K-12 Real World Math Problems
BCAMT	http://www.bcamt.ca/category/weekly-primary-tasks/	http://www.bcamt.ca/weeklymathtasks/
NCTM Illuminations	https://illuminations.nctm.org/	K-12 Searchable (by grade/level) & strand. Provides mini-lessons.
Math Pickle	http://mathpickle.com/organized-by-grade/#	K-12 mathematical puzzles, games and problems
Problem of the Day Archive – Waterloo	http://www.cemc.uwaterloo.ca/resources/potw.php	Grades 3-12
NZ Maths	https://nzmaths.co.nz/problem-solving	K-12 problem-solving lessons and tasks
Dan Meyer	http://blog.mrmeyer.com/category/3acts/	Grades 4+ Click on 3-Act Tasks for problems
Peter Liljedahl	http://www.peterliljedahl.com/teachers/numeracy-tasks	K-12 Collection of numeracy tasks co-constructed with Peter Liljedahl and piloted by B.C.teachers f

Math Wire	http://www.mathwire.com	K-8 Click on Topics>Problem Solving. Lots of other resources
Jo Boaler	https://www.youcubed.org/tasks/	
NCTM Problem of the Week (membership required)	http://www.nctm.org/Classroom-Resources/CRCC/Problems-of-the-Week-Resources/	
Inside Mathematics	http://www.insidemathematics.org/problems-of-the-month/download-problems-of-the-month	videos, lessons, problems of the month, and resources all on math
G Fletchy 3 Act Tasks	https://gfletchy.com/3-act-lessons/	Math tasks that consist of three parts: Act One: a provocation, such as a picture and a question; Act Two: a video with further information; Act Three: the reveal.
Estimation 180	http://www.estimation180.com	estimation challenge
Math for Love	https://mathforlove.com/lesson-plan/	A framework that provides opportunity for differentiation, personal thinking and strategies, and opportunities for <i>all</i> students to share, listen, and communicate their ideas when problem-solving.
Andrew Stadel	https://docs.google.com/spreadsheets/d/19sms4MpuA0071o4qFPJyVKK-OGlnNegMgSL6WAwldb8/edit - gid=0	Gr 6-12 images are designed to start robust discussion
Balanced Assessment	http://hgse.balancedassessment.org/tasks/alpha.html	Spreadsheet of his 3-acts, 180's and other activities
Three-Act Math Tasks	https://tapintoteenminds.com/3act-math/	Three Act tasks and real world math problems.

Classroom Chef	http://www.classroomchef.com/menu/	K-12 math tasks contextual, visual, and concrete
Daily Desmos	http://dailydesmos.com	Intriguing activities designed to build graphing skills
Mathalicious	http://www.mathalicious.com/lessons	K-8 lessons – free and for purchase
Expeditionary math	http://www.expeditionarymath.com	Gr 3+ lessons, wonders, explorations to discover
Hungry Teacher	https://hungryteacher.com/lessons/	real-world problems or questions that need solutions
Math Arguments	http://matharguments180.blogspot.ca	Real World Math Activities that promote student engagement, collaboration, and problem solving
Math Solutions	https://mathsolutions.com/books-resources/classroom-lessons-old/	180 Days of Ideas for Discussion in Math Class
Math World Liason	https://lanewalker2013.wordpress.com/category/3-acts/	Grades 6-12
Numberless Word Problems	https://bstockus.wordpress.com/numberless-word-problems/	
Play With Your Math	https://playwithyourmath.com	K-6 questions designed to have students wonder about numbers
Inside Mathematics	http://www.insidemathematics.org/problems-of-the-month/download-problems-of-the-month	

Literature Based Lessons

What could this look like?

Math in literature ~ Intermediate Selected by Debbie Nelson & Joan Pearce <i>Big Idea : Numbers</i>	
	<p><u>Sisters & Brothers: sibling relationships in the animal world</u> by Steve Jenkins and Robin Page A resource filled with math and numbers about siblings. Identical quadruplets (multiples), family groupings (place value from hundreds, thousands or even millions) At the back of the book you will find out more about each creature's size, what it eats and where it lives.</p>
	<p><u>Frank and Lucky Get Schooled</u> by Lynne Ray Perkins Lucky didn't eat Frank's homework. Lucky was Frank's homework. Perkins uses the school subject areas as a device to highlight all the concepts and facts that Frank and Lucky explore while having fun together. For example: wondering about skunks leads to learning about chemistry; sharing a bed involves proportions and fractions; leaving a chair pulled out from a table that holds a cake prompts an historical investigation; observing a silhouette against the horizon leads to art and a study of perspective; and swimming, fields, and snails are all about geography, of course.</p>
	<p><u>A Hundred Billion Trillion Stars: Can you imagine so many...of anything?</u> By Seth Fishman A book to help student's wrap their brains around large numbers. By comparing things that children can relate to, such as people to ants, this book will help students begin to see the relationships between huge numbers. Co-create and post ideas: What things come in big numbers?</p>
	<p><u>Ten Times Bigger</u> by Richard Michelson Essentially the animals in this book try to best each other...one animal has an amazing attribute but there is always 'ten times better' (more, bigger, faster) than the one who started the bragging. https://letsplaymath.files.wordpress.com/2012/07/multiplication-matching-cards.pdf</p>

Campbell River/Comox

<https://portal.sd71.bc.ca/group/17lwzsl/intermediatemath/Documents/Math%20Bibliography%20Intermediate%202017.pdf>

<https://mathbookmagic.com/>



The BIG Mathematical Idea(s)

The students will UNDERSTAND

Number represents and describes quantity. Developing computational fluency comes from a strong sense of number: Flexible decomposing and composing of numbers are used when adding, subtracting, multiplying, and dividing whole numbers.



Thought provocation:

What other equal groups can you put the penguins in?

Curricular Competencies

The students will BE ABLE TO:

Reasoning and analyzing

- Estimate reasonably
- Develop mental math strategies and abilities to make sense of quantities
- Use reasoning and logic to explore and make connections

Understanding and solving

- Use multiple strategies to engage in problem solving (e.g., visual, oral, role-play, experimental, written, symbolic)
- Develop, construct, and apply mathematical understanding through role-play, inquiry, and problem solving

Communicating and representing

- Communicate in many ways (concretely, pictorially, symbolically, and by using spoken or written language to express, describe, explain, and apply mathematical ideas)
- Describe, create, and interpret relationships through concrete, pictorial, and symbolic representations

Connecting and reflecting

- Visualize and describe mathematical concepts
- Connect mathematical concepts to each other and make mathematical connections to the real world
- Share and reflect upon mathematical thinking

Mathematical Concepts

The students will KNOW:

Grade Three

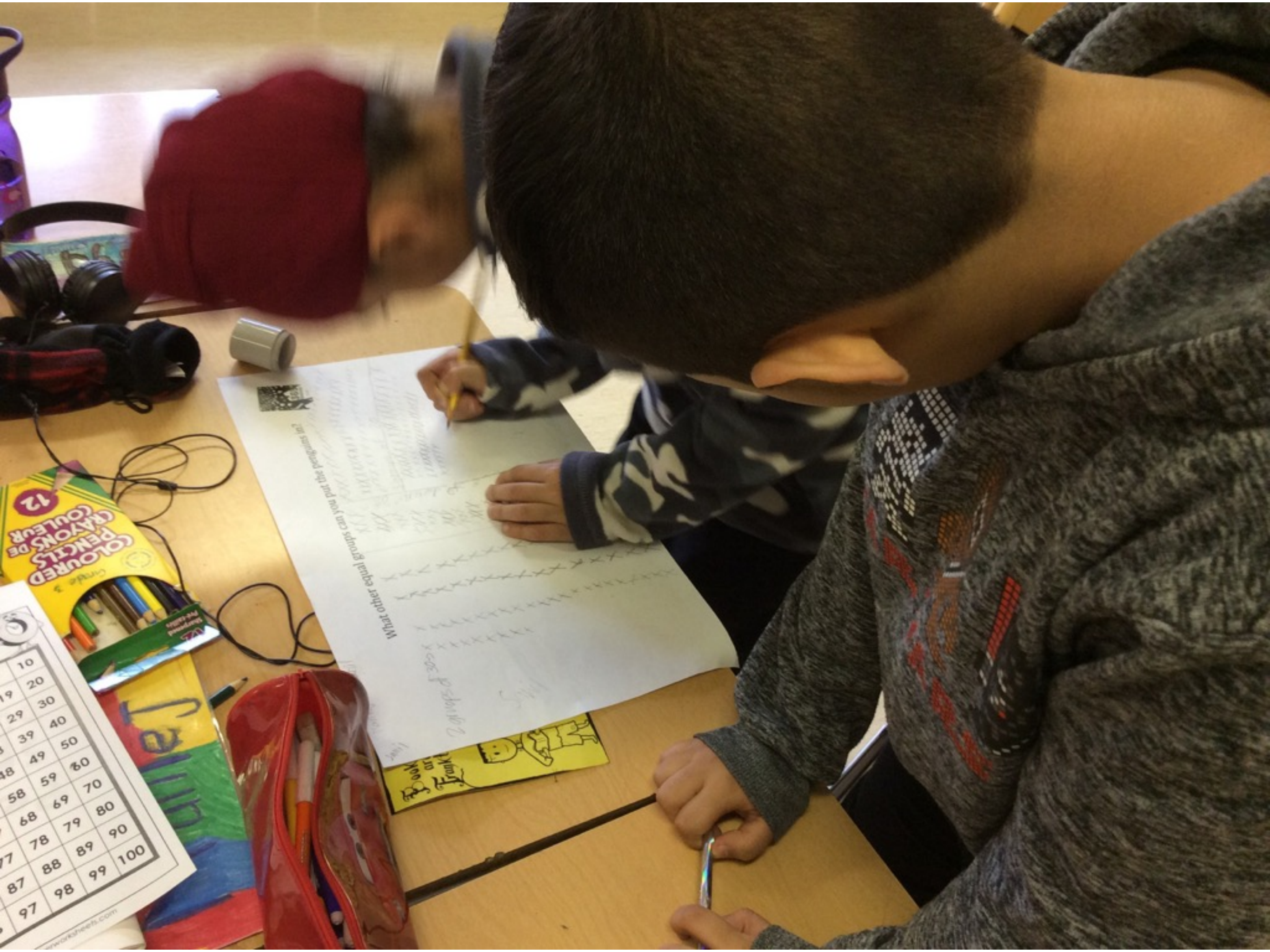
- Number Concepts to 1000
 - counting
 - quantities can be arranged, ordered and compared
- Addition and subtraction to 100
 - Decomposition of numbers to 1000
 - Using strategies such as doubles
- Multiplication and Division concepts
 - opportunities for concrete and pictorial representations of division

What other equal groups can you put the penguins in?



of ten.





10		
19	20	
29	30	
39	40	
48	49	50
58	59	60
68	69	70
77	78	79
87	88	89
97	98	99
		100

What other equal groups can you put the penguins in?



20

XXXXXX
XXXXXX
XXXXXX

20

XXXXXX
XXXXXX
XXXXXX

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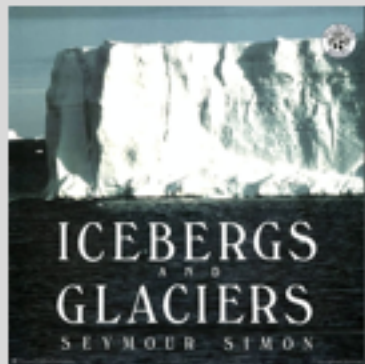
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Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



The BIG Mathematical Idea(s)

The students will UNDERSTAND

Numbers represents and describes quantity: Parts of wholes can be represented by fractions or decimals. We can describe, measure, and compare spatial relationships: Polygons are closed shapes with similar attributes.

Inquiry Question:

Using the plasticine and a piece of string, how might you represent a typical iceberg that has $\frac{1}{8}$ above the water and $\frac{7}{8}$ below the water?

Curricular Competencies

The students will BE ABLE TO:

Reasoning and analyzing

- Estimate reasonably
- Develop mental math strategies and abilities to make sense of quantities
- Use reasoning and logic to explore and make connections

Understanding and solving

- Use multiple strategies to engage in problem solving (e.g., visual, oral, role-play, experimental, written, symbolic)
- Develop, construct, and apply mathematical understanding through role-play, inquiry, and problem solving
- Engage in problem-solving experiences that are connected to place, story, and cultural practices relevant to the local community

Communicating and representing

- Communicate in many ways (concretely, pictorially, symbolically, and by using spoken or written language to express, describe, explain, and apply mathematical ideas)
- Describe, create, and interpret relationships through concrete, pictorial,

Mathematical Concepts

The students will KNOW:

Grade Three

- number concepts to 1000
- fraction concepts
- addition and subtraction to 1000
- measurement using standard units (linear, mass, and capacity)
- construction of 3D shapes

Grade Four

- number concepts to 10 000
- decimals to hundredths
- regular and irregular polygons

Created by JBarker

and symbolic representations

- Use technology appropriately to explore mathematics, solve problems, record, communicate, and represent thinking

Connecting and reflecting

- Visualize and describe mathematical concepts
- Connect mathematical concepts to each other and make mathematical connections to the real world (e.g., in daily activities, local and traditional practices, the environment, popular media and news events, cross-curricular integration)
- Share and reflect upon mathematical thinking
Draw upon local First Peoples knowledge and/or expertise of local Elders to make connections to mathematical topics and concepts

Description of Learning Activity

Every student should be able to show and communicate their understanding of the concepts, and be allowed to represent their understanding through concrete materials, pictures, numbers or words. Providing the opportunities for students to show what they know in a way that makes sense to them is a critical component. Ensure that the manipulatives and 'thinking tools' are accessible. Consider some guiding questions you might ask to scaffold or extend thinking.

Before

- Have a class discussion about what they know about icebergs. Read aloud a non-fiction book on icebergs, paying close attention to images that reflect the typical iceberg. Point out that on average (discuss this term) we can only see $1/8^{\text{th}}$ of an iceberg, as $7/8^{\text{th}}$ is beneath the water. Ask the students to visualize what this might look like. Allow the students to work individually, in partners, or small groups and provide each with a similar sized ball of plasticine and piece of string.

During

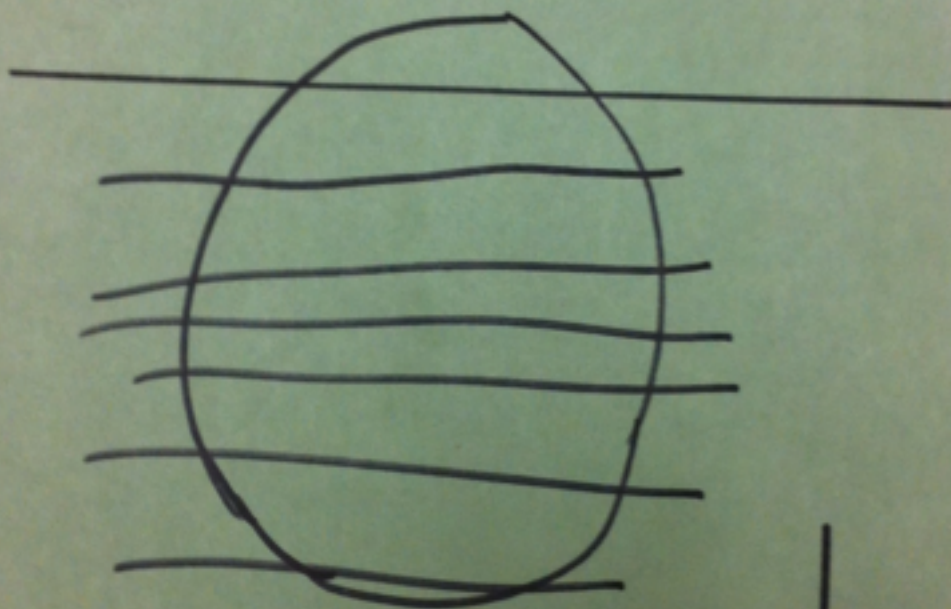
- The teacher will read aloud the Inquiry Question and ask students to discuss how they might approach this question. The teacher will allow students to choose whether to work individually, in pairs, or small groups to solve the question. Each student/pair/group will be provided with a typed copy of the question on an 11 x 17 piece of paper.
- The teacher will provide plasticine and small piece of string, as well as pencils and erasers.
- The teacher will circulate, assist students, and ask prompting questions so as to nudge their learning forward.

After

- Students will take a gallery walk around the classroom pausing at each other's work and listening to each other explain

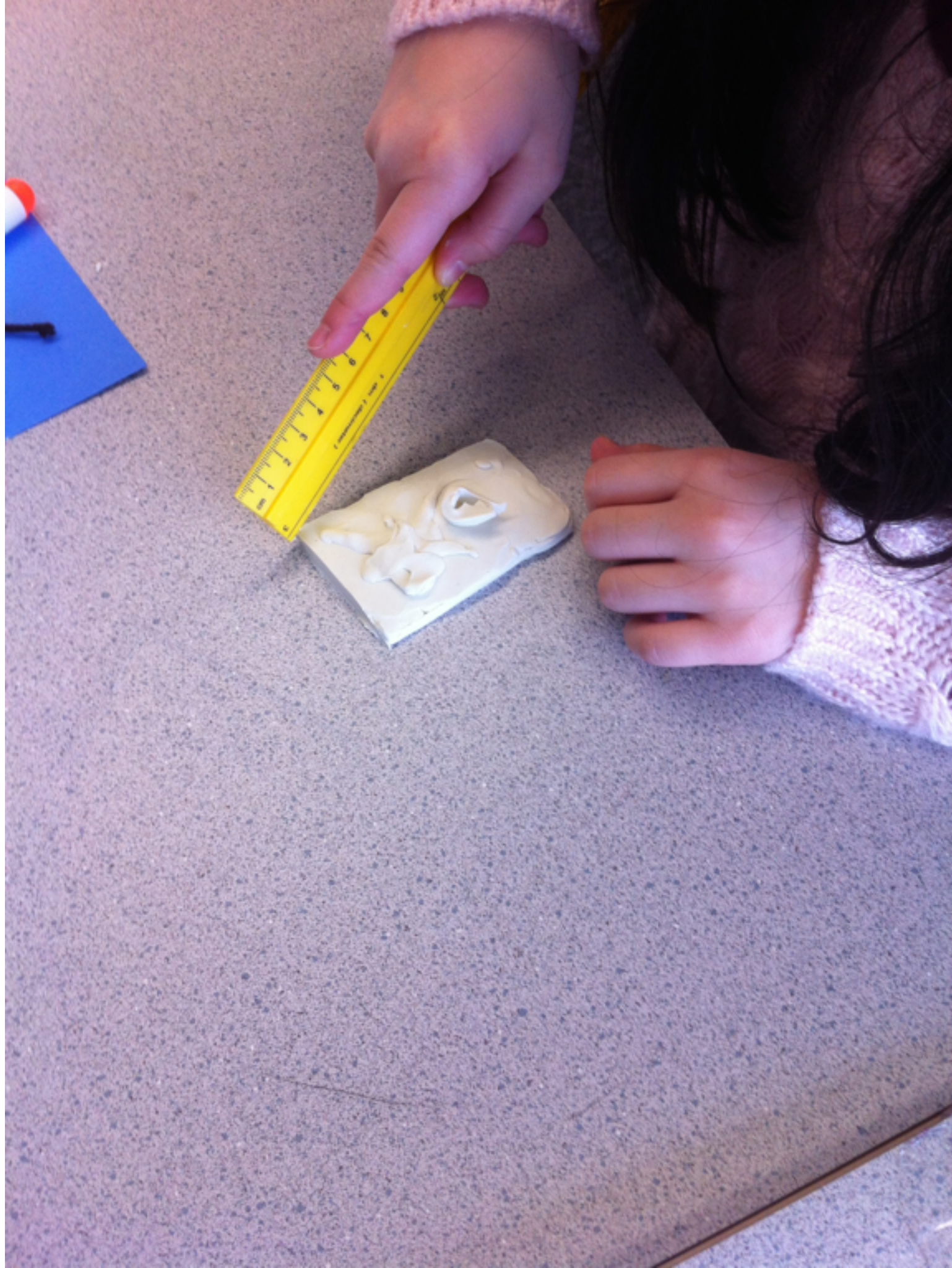
$\frac{1}{8}$ on top

$\frac{7}{8}$ on bottom



8 on


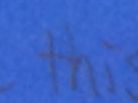




$\frac{7}{8}$

8



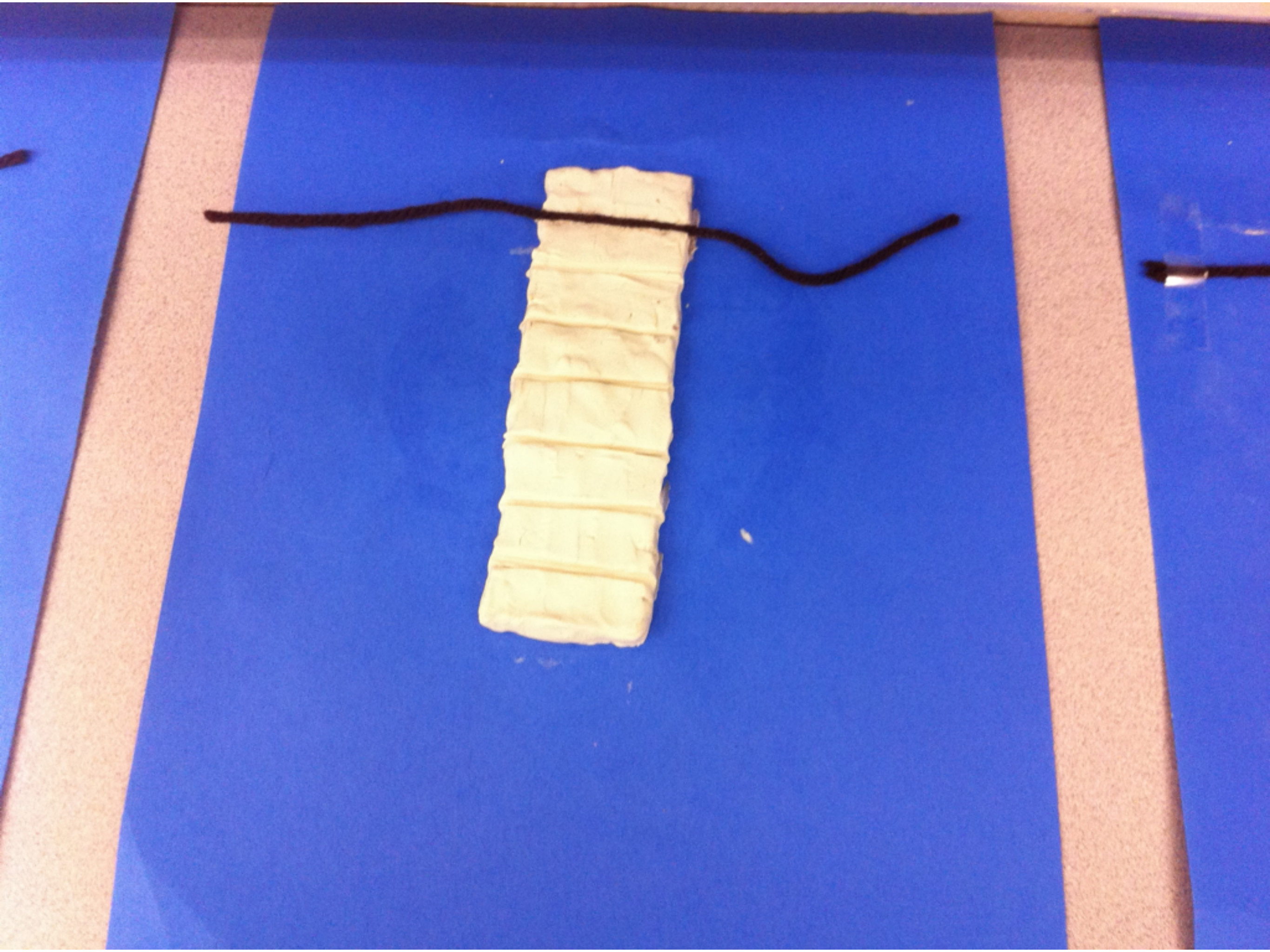
First we made a rectangle. then we madeed this way () it wouldn't work because the ocean it's staight not like this (). So we start all over again.

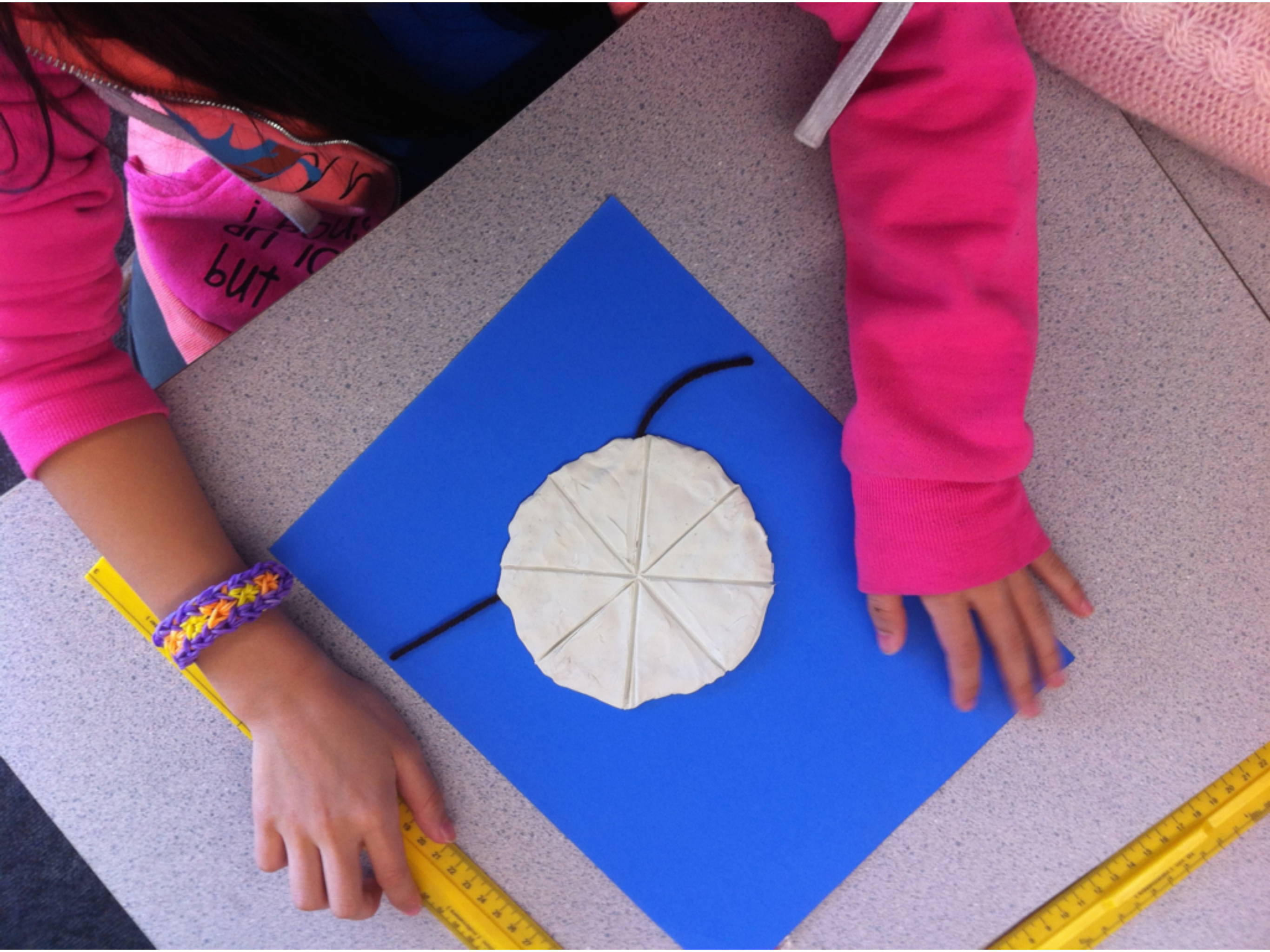
Steps

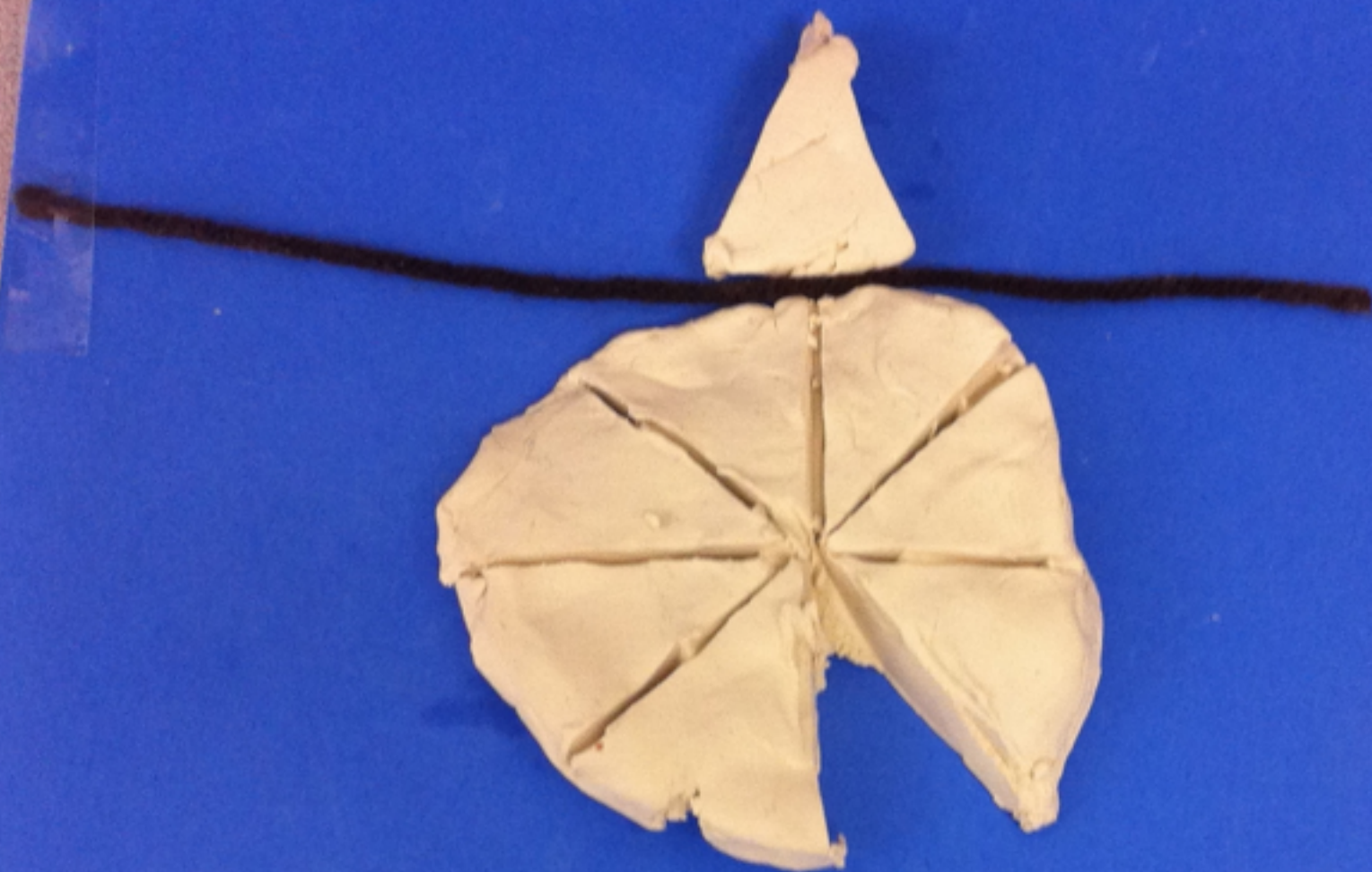
1. Made triangle
2. Cut it half and make 8 equal pieces
3. David steped on it to make it cooler and 3D.

Our strategy was to first use a ruler to spread the plastic to 16 cm. Then we mark every 2 cm until we get to the end. Finally, we cut off one of the 2 cm because every 2 cm is









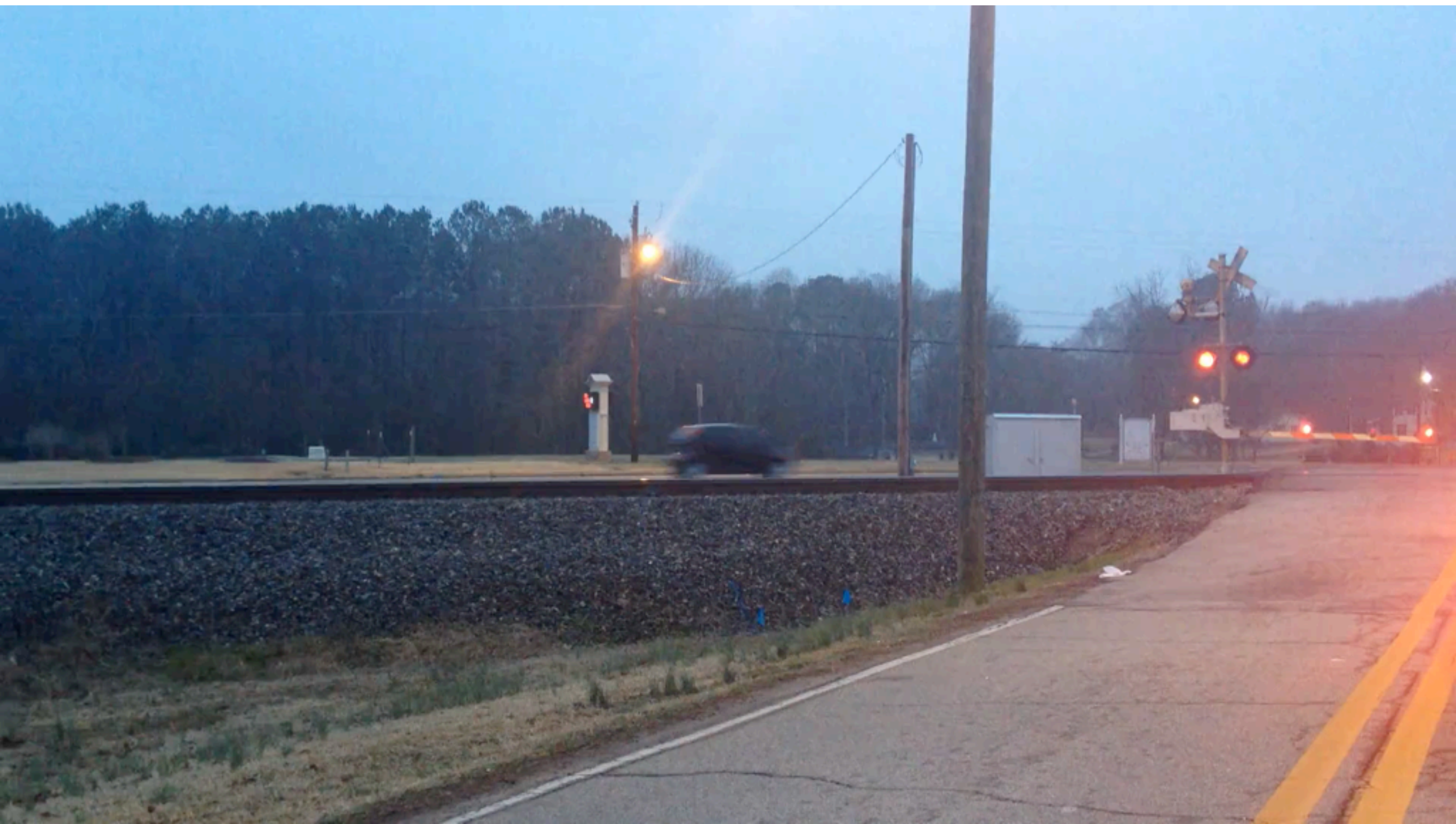
Three Act Tasks

1) The Question, 2) Gathering Information, and 3) The Reveal.

The entire activity typically takes a full math period or the acts can be split up and worked on across multiple days. The goal of the activity is to engage children in asking mathematical questions, identifying information that will allow them to answer the question, developing a mathematical model of the situation, and revising their models to more closely reflect the real world.



Act One



Act Two

There's 70 train cars.

There's 2 locomotives.

It takes 10 seconds for 7 train cars to pass.

It takes 3 seconds for 1 locomotive to pass.

Act Three



Which would you rather? This

Word Problems

- 1) Sara had 12 Pokemon cards. Benny gave her 23 new Pokemon cards. Sara bought 22 Pokemon cards. How many Pokemon cards does Sara have now ?

- 2) Joan went to 19 soccer games this month. She went to 11 games last month, and plans to go to 13 games next month. How many games will she attend in total ?

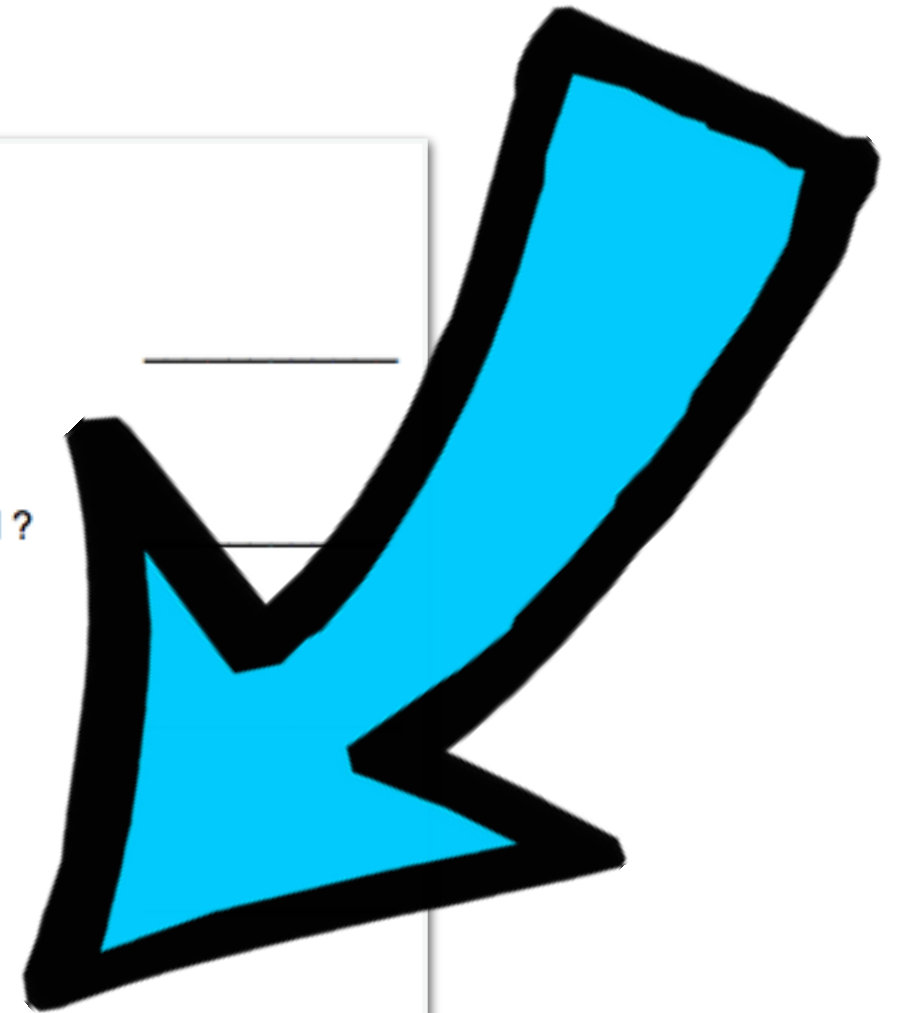
- 3) Sally has 36 red balloons, Tim has 21 red balloons, and Jessica has 35 red balloons. How many red balloons do they have in all ?

- 4) Fred picked 17 pears, Jason picked 29 pears, and Melanie picked 12 pears from the pear tree. How many pears were picked in all ?

- 5) Sara found 27 seashells, Fred found 19 seashells, and Nancy found 36 seashells on the beach. How many seashells did they find together ?

- 6) Sara had 37 nickels in her bank. Her dad gave her 32 nickels and her mother gave her 43 nickels. How many nickels does Sara have now ?

- 7) Alyssa has 49 books, Dan has 25 books, and Jason has 36 books. How many books do they have together ?



Or tasks, like the one we just did where students are:

- making sense and understanding the context
- asking interesting questions which math can solve
- determining what information is needed to solve the problem
- mathematically modelling situations

Website 3-Act Tasks (Graham Fletcher) ☆ 🌐

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	A	B	C	D	E	F
	Date Added	Lesson Title	Standard 1	Standard 2	Big Ideas	What do you wonder?
1	4/17/2014	Peas In a Pod	K.NBT.1	K.CC.4	counting	If all the peas were in one pod, how many peas would there be?
2	4/25/2014	Dotty	K.CC.1,2,3	K.CC.4,5	counting and patterns	How many dots will be on the screen after the last bell?
3	2/9/2016	the Candyman	K.CC.1,2,3	K.CC.4,5	counting and joining sets	How many candies are in are in his hand?
4	12/6/2015	Share the Love	K.CC.1,2,3	.	sharing quantities within 20	How many M&Ms will each girls get?
5	1/16/2015	Counting Squares	K.NBT.1	K.CC.4,5	counting and patterns	How many tiles are in the pile?
6	1/16/2015	Stage 5 Series	K.NBT.1	K.CC.4,5	counting and patterns	What will stage 5 look like?
7	3/24/2015	Shark Bait	K.NBT.1	K.CC.4,5	counting and joining sets through 20	How long is the worm?
8	3/4/2014	Lil' Sister	K.MD.2	K.CC.6	comparing measurements	How much shorter is Lil' Sister than Big Sister?
9	9/1/2015	Bag-O-Chips	K.OA.4	K.OA.5	building fluency through 10	How many bags of chips were missing?
10	5/8/2014	Balancing Numbers	K.OA.2	.	number combinations through	What is needed to make both side of the scale equal? (balance)
11	9/27/2015	Humpty Dumpty	K.OA.1,2,3	.	addition and subtraction within 20	How many eggs didn't break?
12	10/10/2017	Popping Balloons	K.OA.1,2,3	.	building fluency through 10	How many balloons are left?
13	2/15/2015	the Cookie Monster	1.NBT.1	1.NBT.4	addition and subtraction within 50	How many cookies did the cookie monster eat?
14	11/7/2016	the Pringle Ringle	1.NBT.1	1.NBT.4	addition and subtraction within 100	How many Pringles did it take to make the Pringle Ringle?
15	5/3/2014	the Juggler	1.NBT.1	1.NBT.4	addition and subtraction	How many times will the juggler be able to bounce the ball off a body part unt
16	11/10/2014	Graham Cracker	1.NBT.1	1.NBT.4	addition and subtraction within 100	How many crackers will fit inside the Graham Cracker box?
17	5/16/2016	Bright Idea	1.NBT.1	1.NBT.4	addition and subtraction within 100	How many Skittles fit inside the light bulb?
18	9/4/2017	Snack Machine	1.NBT.6	.	addition and subtraction within 100	How much did the Munchos cost?
19	3/30/2017	Sliced Up	1.G.3	4.NF.4	working with quarters and wholes	How many orange wednes are in the bowl?
20	2/9/2016	the Whopper Jar	2.NBT.5	1.NBT.4	addition and subtraction within 100	How many Whoppers are inside the jar?
21	3/3/2014	Cover the Floor	2.OA.4	.	building arrays with repeated addition	How many blue squares will it take to cover the yellow square?
22	3/7/2015	It All Adds Up	2.NBT.5	.	adding and subtracting money	What coins are in the bank?
23	9/9/2015	Let It Fly	2.NBT.7	.	adding and subtracting within 1000	How far did he throw the disc?
24	2/1/2016	Downsizing Tomatoes	2.NBT.7	.	adding and subtracting within 1000	How many little ketchup bottles will will the big bottle fill up?

to learn more....

Three-Act Tasks



ABOUT THIS ACTIVITY

This activity is made up of three parts or "acts;" 1) The Question, 2) Gathering Information, and 3) The Reveal. The entire activity typically takes a full math period or the acts can be split up and worked on across multiple days. The goal of the activity is to engage children in asking mathematical questions, identifying information that will allow them to answer the question, developing a mathematical model of the situation, and revising their models to more closely reflect the real world.

INTRODUCE

PREPARE

ENACT

ANALYZE



ABOUT THIS ACTIVITY	
INTRODUCE	
<h1>1</h1> <h2>Introduce</h2> <p>Introducing Practices of Ambitious Teaching</p>	<hr/> <h3>Resources</h3> <ul style="list-style-type: none"> Three-Act Task Primer Article: Trying Three-Act Tasks with Primary Students Three-Act Task Overview Modeling with Mathematics Primer Blog Post: Modeling with Mathematics Video: Three-Act Task in Kindergarten Video: Three-Act Task in Second Grade



ABOUT THIS ACTIVITY	
INTRODUCE	
PREPARE	
<h1>2</h1> <h2>Prepare</h2> <p>Helping Teachers to Prepare to Enact</p>	<hr/> <h3>Resources</h3> <ul style="list-style-type: none"> Planning Guide Planning Template Three-Act Task Resources Anticipating Student Thinking Three-Act Task Planner



ABOUT THIS ACTIVITY
INTRODUCE
PREPARE
ENACT

3

Enact

Supporting Teachers to Enact Ambitious Teaching

Resources

- [Planning Template](#)
- [Student Recording Sheet \(version A\)](#)
- [Student Recording Sheet \(version B\)](#)



ABOUT THIS ACTIVITY
INTRODUCE
PREPARE
ENACT
ANALYZE

4

Analyze

Guiding Teachers in Analyzing Practice

Resources

- [Prompts for Reflection](#)

When does Math feel like PLAY to you?



Playful Mathematical Inquiry

Begin with your students interests and curiosities...

Think like a child...

What materials will pique curiosity and act as an invitation?

Discuss what you notice and wonder

Provocations can include:

- Books
- Artifacts
- Photos



Image from Janice Novakowski - http://janicenovkam.typepad.com/reggioinspired_mathematic/

Types of Inquiry

Structured Inquiry

- The teacher determines the big idea and what the students will come to understand by the end.
- The teacher starts with a guiding question.
- The students will help create the plan and guide the inquiry with their questions, interests, ideas, analysis, reflections, and understandings.

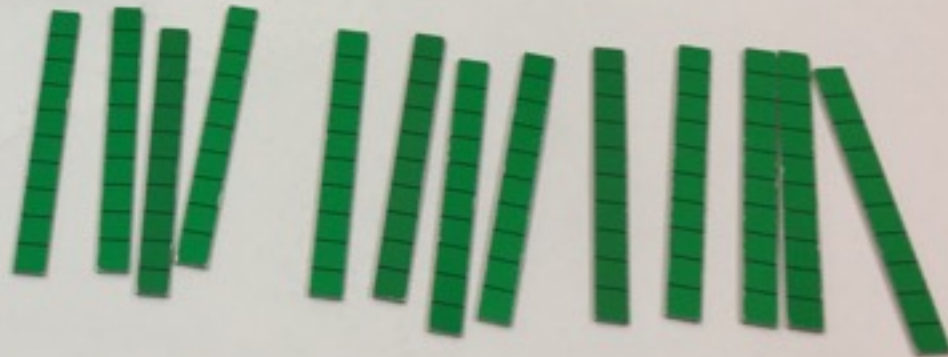
Guided Inquiry

- The teacher comes up with the big idea or topic and students and/or the teachers come up with the questions.
- The students are responsible for designing and following their own procedures to test the question and then communicate their results and findings.

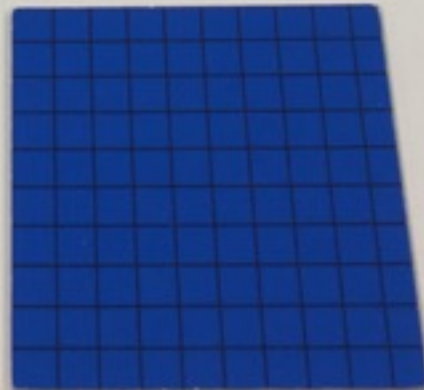
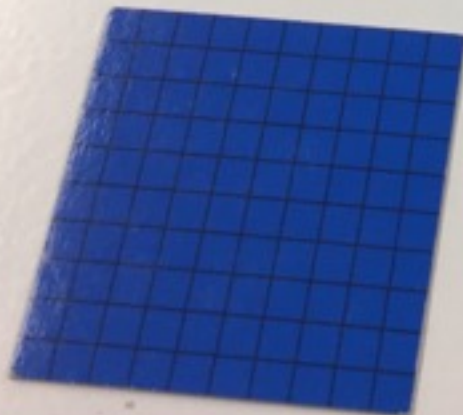
Number Concepts:

You use base 10 blocks to represent a number. Can it take fewer blocks to represent a greater number?

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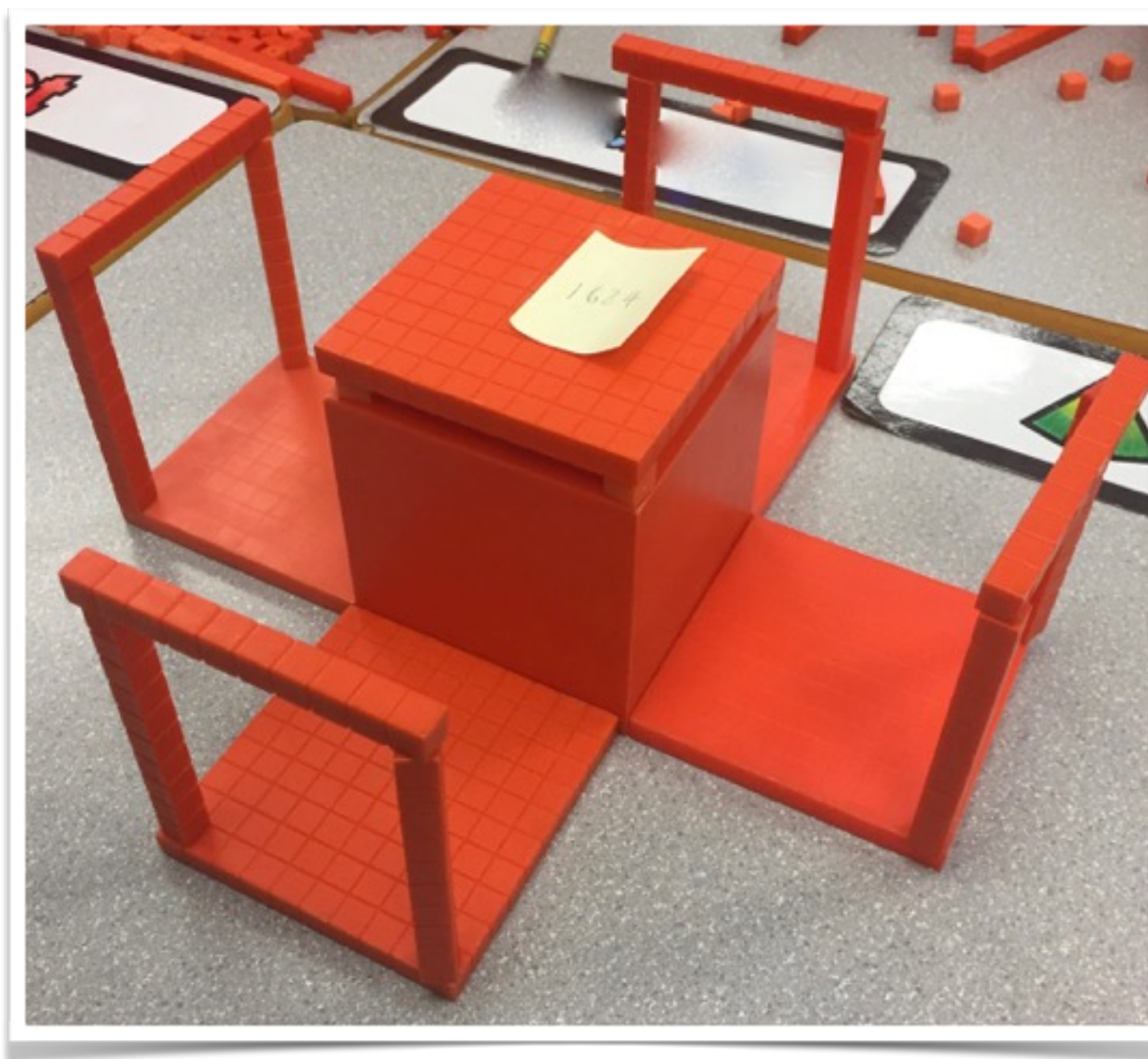


232



Choose a 4-digit number.

How many different ways can you represent your number with base 10 blocks?



How do tables and charts help us to understand patterns?



What colour will the 25th cube be? Or the 100th?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Patterning:

How do tables and charts help us understand patterns?

Steps	Pumpkins
1	1
2	3
3	5

Patterning:

Can you represent expressions?

Can a pattern be made with items that are all the same (e.g., size and shape)?

$$n + 1$$



$$2n$$



$$3n + 1$$



PLAY GIVES
CHILDREN
A CHANCE
TO PRACTICE WHAT
THEY ARE
LEARNING.
-MR. ROGERS

Independent Practice

Provides authentic, engaging learning opportunities to review competencies/concepts. Students will need to take ownership of their own learning and work independently or with a partner to develop their understanding.

Considerations:

- Provide learning opportunities that will allow students to apply understanding and share thinking.
- Ensure that learning opportunities are CAREFULLY selected to focus on the learning intentions and provide STUDENT CHOICE
- Ensure there are diverse learning opportunities that provide ACCESS FOR ALL.
- Provide games, rich tasks, or provocations that involve thinking, reasoning, and communicating which assist students in developing their understanding of mathematics.

Textbooks

Remember one size does NOT fit all!

2. Write the next 3 numbers in each pattern.

a) 6, 12, 18, 24, ... c) 5, 15, 25, 35, ...
b) 49, 42, 35, 28, ... d) 1, 3, 5, 1, 3, 5, 1, 3, 5, 1, ...

3. Describe each pattern. Write the next 3 numbers.

a) 25, 27, 29, 31, ...
b) 0, 1, 3, 4, 6, 7, 9, ...
c) 22, 19, 16, 13, ...
d) 0, 15, 30, 45, 0, 15, 30, 45, 0, 15, 30, 45, ...

4. A pattern starts at 60 and then decreases by 1, then 2, then 3, and so on. What is the 10th number in the pattern? Show your work.

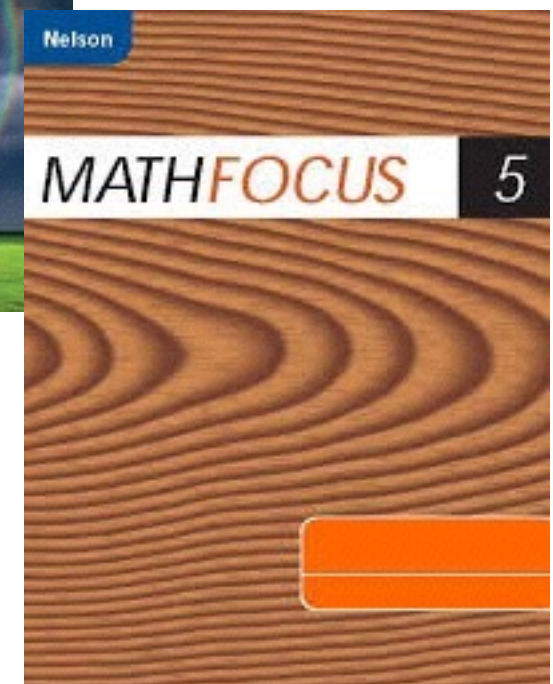
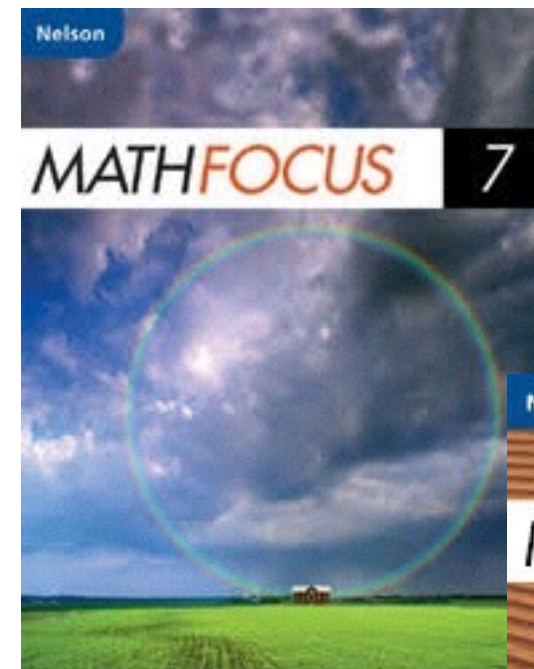
5. The library gets 16 new books each month.

a) Complete the t-chart to show how the number of new books grows over 1 year.
b) Write a rule for the pattern in the 2nd column of the t-chart.
c) Describe any patterns in the digits. Predict the next 3 numbers.

Month	Total number of new books
1	

6. It takes Suki about 45 minutes to make a bracelet. Make a t-chart that shows how many bracelets she can complete in 4 hours.

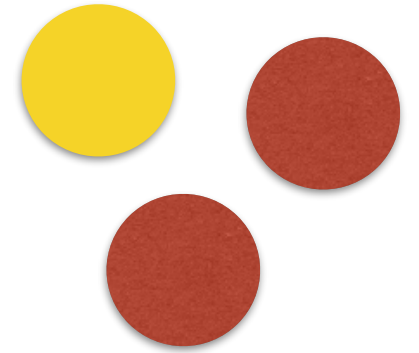
12





Independent Practice

What could this look like?



 **Double or Double-Double**


Pick a factor from the Factor Box.
Double it (multiply by 2) or double-double it (multiply by 4).
Find the product below and cover it in your colour.
Four in a row wins!

1	2	3	4	5	6
7	8	9	10	12	14

24	8	6	20	12	4
16	28	14	32	18	24
10	2	6	36	28	40
12	14	18	4	16	20
10	8	24	32	36	2
40	28	18	16	4	6

Cover Up! A Doubles Game


How to Play
Roll a 10-sided die. Multiply your number by 2.
Find it on the grid and cover it in your color.
Partner 2 gets a turn.
The first one to have 4 in a line wins!
Roll a 0 and lose a turn!



You need:
- 10 sided dice
- Counters (2 different colors)

12	6	10	12	4	8
14	18	6	16	2	6
6	10	2	12	10	14
12	4	8	18	16	10
14	16	2	14	4	12
10	8	4	2	16	18

Four in a Line for 2 players



You need:
- a pencil
- counters in two colors
- a calculator

When it's your turn
Choose a number from the box.
Double it or multiply it by 4. Put a counter of your color on the answer on the grid.
If you want, mark which number you used—though you may use it again later.
The end of the game
Go on playing like this until one of you has made a line of four counters in your color. That person wins.

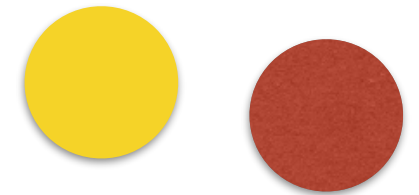
12	39	75	370
16	40	63	450
26	42	55	500
33	45	400	
35	50	420	

90	48	1000	32	70	80
2000	52	110	78	150	66
220	900	252	800	156	64
1800	740	24	140	840	132
126	200	300	100	180	1680
104	84	1480	160	168	1600

independent and/or Partner Games



Independent Practice



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7	8	9	10	12	14

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16	28	14	32	18	24
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12	4	8		16	10
14	16	2	14	4	12
10	8	4	2	16	18

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independent and/or Partner Games



Independent Practice

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1	2	3	4	5	6
7	8	9	10	12	14

24	8	6	20	12	4
16	28	14	32	18	24
10	2	6	36	28	40
12	14	18	4	16	20
10	8	24	32	36	2
40	28	18	16	4	6

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6	10	2	12	10	14
12	4	8		16	10
14	16	2	14	4	12
	8	4	2	16	18

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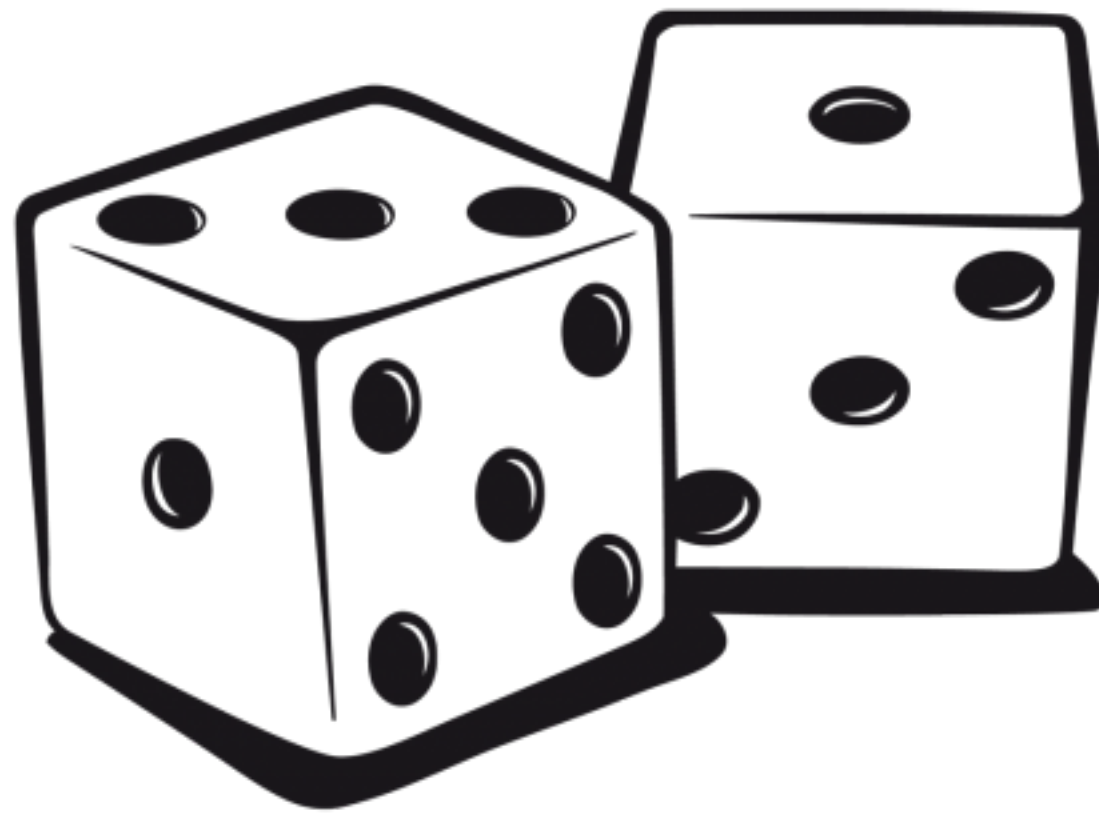
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126	200	300	100	180	1680
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independent and/or Partner Games

Box Cars and One Eyed Jacks



TO SUM IT UP

LEVEL: Grade 3 and up

SKILLS: adding 3 digit numbers

PLAYERS: 2 or more, or teacher vs whole group

EQUIPMENT: cards (Ace=1) - 10 (10=0), one gameboard for each player (see reproducibles)

GETTING STARTED: The object of this game is to make the greatest sum. The deck is placed face down. A card is drawn and is placed face up. Each player selects a space on their gameboard and writes the number of this card in it. Eight more cards are drawn and players proceed to fill in their gameboards. Once all spaces are filled in, players complete the addition. The player with the greatest sum is the winner of that round and scores one point. As players have more experience with this game, they will develop strategies to maximize their chances.

EXAMPLE: First card turned over is a 5. Second card turned over is a 7. Seven more numbers are drawn and completed gameboards could look like this:

Player 1	Player 2	Player 3
3 5 4	8 7 0	9 0 2
7 9 8	5 4 3	6 4 5
+ 0 6 2	+ 9 2 6	+ 8 7 3
1 2 1 4	2 3 3 9	2 4 2 0

Player 3 has the greatest sum and is the winner for this round.

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Independent Practice Time



Pick a “just right” game

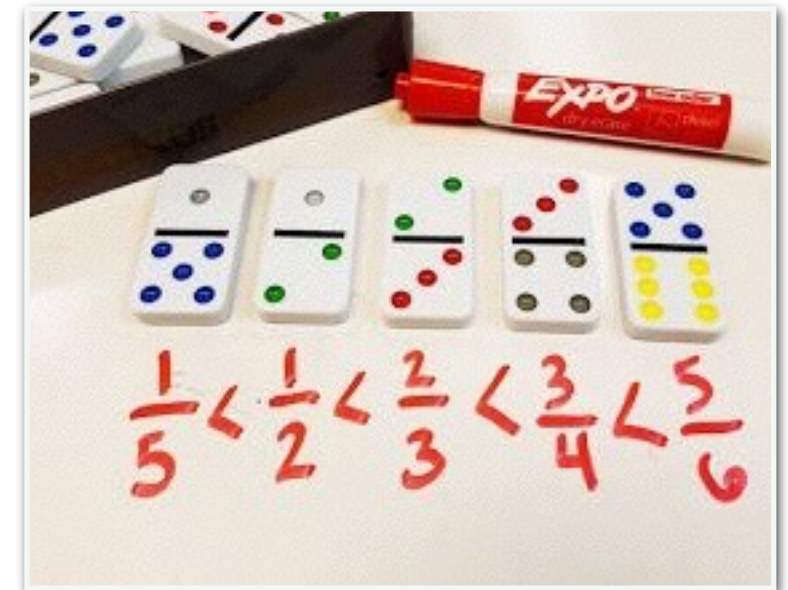
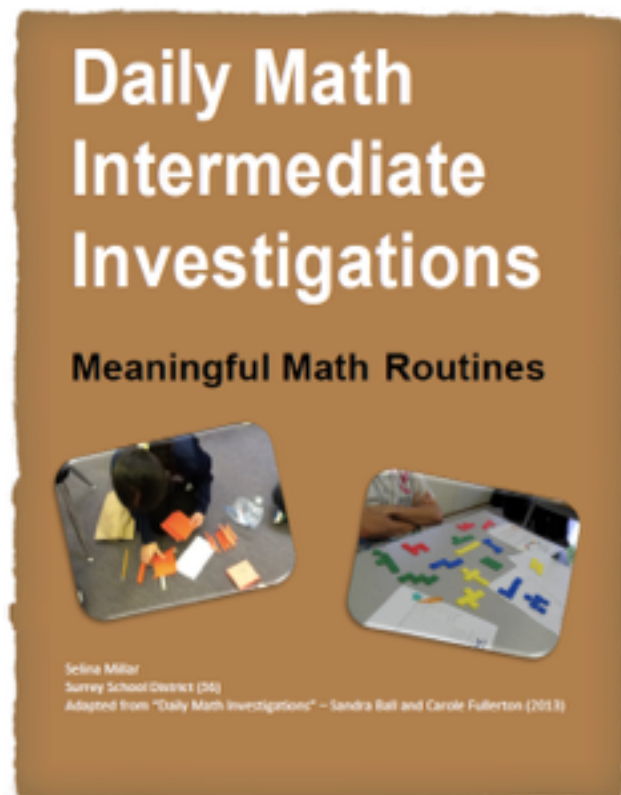
Self-monitoring promotes automaticity with the basic facts. Self-monitoring requires that students focus their attention on some specific aspect of their learning. As students monitor themselves, they think about what they know and what they still need to learn.

- p.g. 77 Math Running Records

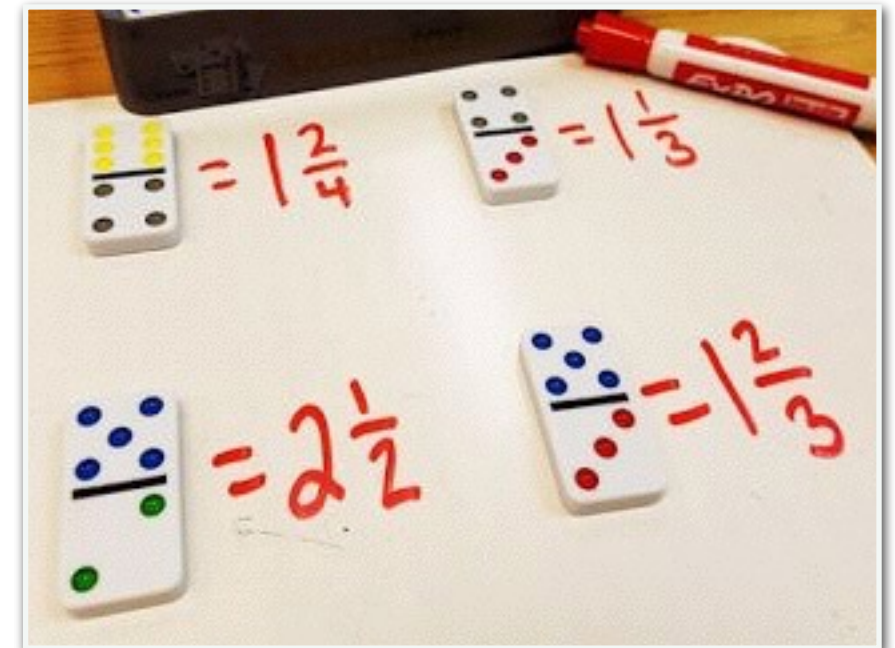


Daily Math Investigations

What does this look like?





- investigations are not new - work in the activities you used in whole class lessons
- students choose where they go
- students can work alone or with others
- each investigation can be differentiated




Level 1

Shape Shifter 4


- Use only 4 of the tangram pieces to make the triangle.
- Move only one of the pieces to form a square.

Make this  Change to this 








You want to provide options for all students; therefore provide opportunities to investigate a number of concepts (e.g., geometry, patterning, number operations, financial literacy)

Number Puzzler





- Use any of the numbers 1-9 to make the given products.
- The across, down, and diagonal products are given as clues.

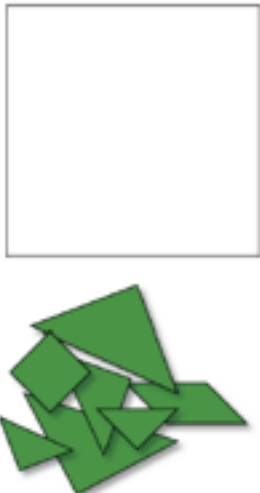
	
	
	

Level 2

Shape Shifter 5

- Use only 5 of the tangram pieces to make the square.
- Move only one of the pieces to form a rocket.

Make this  Change to this 



Level 1

Cards Plus



- Use the cards 1 (Ace) to 9.
- Place the 9 cards in a plus sign as shown to solve one of the problems.
- If you have time, try solving both problems.

Problem #1

Arrange the cards so that they add up vertically and horizontally to:

25



Problem #2

Arrange the cards so that they add up vertically and horizontally to:

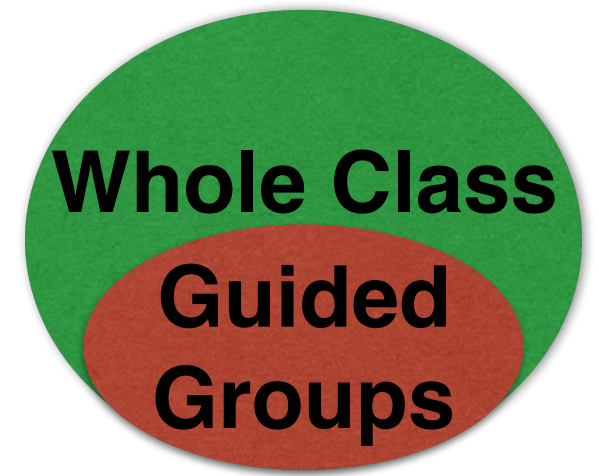
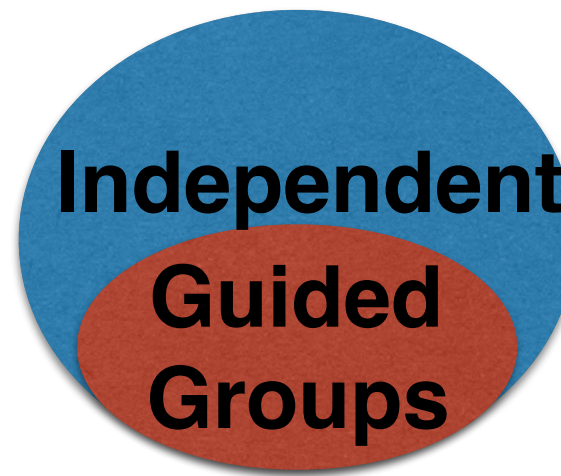
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IMPORTANT

Nested within Independent Practice
and Whole Group Lessons is

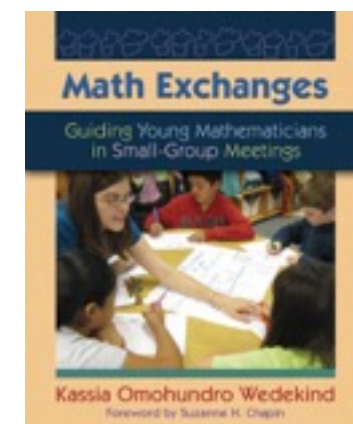


Guided Small Group Instruction

Learning opportunities that support students' strengths and stretches and intentionally move them forward.

- Present rich tasks to identify the students' prior knowledge
- Identify the students' strengths and stretches.
- Focus on scaffolding or extending the learning opportunities.
- Conferencing with the students and asking probing questions.
- Embedded formative assessment to help inform instruction.
- Focus on building students' confidence and risk taking.
- Introduce new learning opportunities for independent practice.

Kassia Omohundro Wedekind author of
<https://www.youtube.com/watch?v=Lgl2EeqApCo>



Guided Small Group Instruction

What could this look like?



- Groups are **FLEXIBLE** and composition changes according to the needs of the students.
- Might include working with students on practice questions, teaching a new game, reviewing a concept taught to the class or working with students who are unsure how to start a problem

Why is it important for teacher's to confer with students?

- Conferring builds identity and agency.
- Deep listening is at the heart of conferring.
- Conferring follows a predictable structure.
 - The teacher asks guiding questions to determine the student's understanding
 - Decide on a point to nudge
- Nudges may connect what a student is doing with ideas being considered by classmates or by the class as a whole.
- A goal of conferring is to facilitate understanding that goes beyond the single problem in front of the student.

Why do we need all 3 parts?

“For a person learning to play baseball, batting practice is an important part of learning how to play the game. However, imagine a person who has never [played] a baseball game. Making that person do nothing but batting practice may lead to the misconception that baseball is about standing at the plate and repeatedly swinging at the ball. That person would miss the purpose of baseball and would think it a boring way to spend an afternoon.” (Stahl, 1992).



What might a week look like?

	Monday	Tuesday	Wednesday	Thursday	Friday
Number Routines (10 - 15 min) These vary depending on my intentions and classroom observations.	<u>Number Talk:</u> Quick Image of an array. Flash and tell how many – how did you see them – can you see it another way? Can we record as an equation?	<u>Number Talk:</u> Present students with a string of questions that will elicit using partial products 2×7 $4 \times 7 = (2 \times 7) + (2 \times 7)$ 4×8 3×8	<u>Notice and Wonder with Data Analysis:</u> Preset graph with no labels – what do you notice and wonder?	<u>Choral Count</u> – Focusing on counting by 100's starting at 346 - 3000	<u>Number Lines</u> – Build the line – show a number – provide quiet thinking time – talk at your talk – where would you place this and why. Add the 5 numbers then provide mystery number.
Whole Class (45 min) Note: Teacher meets with small groups as needed NOTE: The middle part of the 3-part lesson includes students DOING mathematics which may be independent.	Counting Collections Mini-lesson: How might we count the collections using fractions?	Concept based 3- part lesson: Inquiry-based Problem solving Open task Parallel task Three Act Task Text book whole class lesson	Concept based 3- part lesson: Inquiry-based Problem solving Open task Parallel task Three Act Task Text book whole class lesson	Concept based 3- part lesson: Inquiry-based Problem solving Open task Parallel task Three Act Task Text book whole class lesson	Concept based 3-part lesson: Inquiry-based Problem solving Open task Parallel task Three Act Task Text book whole class lesson
Guided Groups (5 - 10 min with each group) This happens during whole class as well as during the independent practice)	While the students are counting the collections, the teacher is circulating and conferencing with small groups.	During the Independent Practice time, the teacher meets with specific students in small groups. Guided instruction and/or explicit.	During the Independent Practice time, the teacher meets with specific students in small groups. Guided instruction and/or explicit.	During the Independent Practice time, the teacher meets with specific students in small groups. Guided instruction and/or explicit.	
Independent Practice (30 min) - allows us to revisit concepts and keep concepts alive all year	While the students are counting the collections, the teacher is circulating and conferencing with small groups.	(30 min) – Soft Start - Daily Math Investigations: centres/games/ choice	(30 min) – Soft Start - Daily Math Invest – centres/ games/ choice	30 min) – Soft Start - Daily Math Invest – centres/ games/ choice	

How might we think about planning for the year?



Find out what your students know -
This doesn't mean a test the first week!



Build a community of learners - what is their history with Math? Provide opportunities to uncover what it means to be a Mathematician.

Begin with a unit in which students can learn about one another - data analysis or how to engage with materials.

Consider including a visual spatial unit each term so that students for whom this is a strength have an opportunity to feel success.

What concepts are new to this grade?

What concepts need to be experienced through out the year?

Thank you for your time!

Math Program

