

Multiplication and Division in Grades 3 - 5

UNDERSTAND: Develop conceptual understanding of multiplication. Develop computational fluency and multiplicative thinking requires analysis of patterns and relations in multiplication. Additionally, computational fluency and flexibility with numbers extends to operations with larger (multi-digit) numbers.

KNOW:

What is the learning trajectory for developing an understanding of Multiplication and Division?

Grade Three - Five

- Understanding concepts of multiplication – single digits only
- Understanding concepts of division – two or three-digits by single digits
- Multiplication and Division are related – up to three digits

The following Critical Learning Phases are from Kathy Richardson's in *How Children Learn Number Concepts: A Guide to the Critical Learning Phases* (2012). In the last phase, I have adapted and included strategies from Sherry Parrish's *Number Talks* (2014).

Recognizing Equal Groups

- Counts by equal groups (e.g., 2's, 5's, 10's, and so forth)
- Knows the quantity stays the same when counted by different sized groups (conservation of number)
- Identifies and extends the number pattern that emerge when counting by equal groups

Multiplying Equal Groups

- Counts groups as single entities (unitizes)
- Distinguishes between number of groups and number of objects in a group
- Show with models "a number of groups of a certain size" when the language of "groups of" is presented with various terms (e.g., "piles of," "stacks of," "rows of," "cups of,")
- Interprets word problems using models and drawings showing the number of groups and the number in each group
- Records number of groups in each step of a skip counting pattern, relating it to multiplication (**Repeated addition and skip counting**)

Multiplying Using Rectangular Arrays

- Builds rectangular arrays using "rows of"
- Describes arrays in terms of equal groups (usually by rows)
- Partitions arrays into smaller arrays
- Describes arrays in terms of equal groups when the array is only partially visible

DO:

What will my students DO to show me their knowledge and understanding?
What curricular competencies do we want to focus on?

Reasoning and Analyzing

- Use reasoning to explore and make connections between multiplication and skip-counting and multiplication and repeated addition
- Develop mental math strategies such as using facts they know to solve facts they don't know
- Model multiplicative situations (e.g., groups of equivalent sets, rate/price/length, rectangular arrays, multiplicative comparison, combination problems)

Understanding and Solving

- Apply mathematical understanding of multiplication through play, inquiry, and problem solving
- Visualize groups of and arrays and other multiplication concepts
- Use multiple strategies to engage in multiplication questions
- Engage in problem-solving experiences that are connected to play, story, and perspectives relevant to local First Peoples communities

Communicating and Representing

- Communicate their understanding of multiplication in many ways including orally, concretely, pictorially, symbolically, in written text and/or using screen-casting apps such as ShowMe.
- Use and interpret mathematical vocabulary (e.g., factors, product, twice as many, per, arrays, groups of)
- Explain and justify their mathematical thinking as it relates to multiplication
- Represent understanding concretely, pictorially, and symbolically

Using Multiplicative Thinking to Solve Single and Multi-Digit Multiplication Problems

- Uses multiplication to solve a problem for at least part of the answer and identifies the number of groups (for that part)
- Interprets and writes multiplication and division questions to describe situations in problems
- Interprets language of “twice,” “times as many”, and “per”
- Uses rectangular arrays to show numbers beyond 100 and beyond; describes the composition of the numbers and determines how many all together
- Applies strategies for unknown facts, including:
 - **Using known facts and compensating** e.g., $7 \times 8 =$
But I know $7 \times 7 = 49$
 $49 + 7 = 56$
 - **Doubling and Halving** e.g., for $50 \times 12 =$
 $100 \times 6 = 600$
 - **Making landmark or friendly numbers** e.g., for 9×7
+ 1

 $10 \times 7 = 70$
 $70 - 7 = 63$
 - **Using the commutative property** (e.g., $3 \times 4 = 4 \times 3$)
 - **Using the distributive property** e.g. 12×15
 $12 \times (10 + 5) =$
 $12 \times 10 = 120$
 $12 \times 5 = 60$
 $120 + 60 = 180$
 - **Using the associative property** e.g., $12 \times 36 =$
 $12 \times (6 \times 6) = (12 \times 6) \times 6$
 $72 \times 6 = 432$

Connecting and Reflecting

- Reflect on mathematical thinking
- Connect repeated addition and skip counting to multiplication
- Connect multiplication and division
- Connect multiplication to application in the world
- Incorporate First Peoples worldviews and perspectives to make connections to mathematical concepts

Acknowledgements: These ideas have been collected and/or inspired from a number of sources, including: Marilyn Burns Teaching Arithmetic Series; Kathy Richardson's How Children Learn Number Concepts book and her Developing Number Concepts series; Jo Boaler's Mindset Mathematics; Carole Fullerton's Multiplicative Thinking and Remarkable Cuisenaire Rods; Julie Dixon & others Making Sense of Mathematics For Teaching Grades 3 – 5; John Van De Walle's Student Centered Thinking series, Marian Small's Open Questions, Chris Hunter's Paint Splatter Arrays, Janice Novakowski's Reggio-Inspired work, Michelle Hikida's Mathematical Inquiry session, Sherry Parrish's Number Talks, and Making Number Talks Matter by Cathy Humphreys and Ruth Parker.

How will I find out what my students already know? How will I activate their prior knowledge?

This lesson comes from Marilyn Burns (2001) Teaching Arithmetic: Extending Multiplication, pg 1 - 14.

Although an older lesson, this is a great lesson to discover what your students know. It provides the formative assessment that is needed to plan a responsive multiplication unit. You will begin by letting the students know that the purpose of this lesson is for you to find out what they know about multiplication, as well to develop their understanding of multiplication.

Write the question 8×7 (adapt these factors based on your knowledge of your students) and ask the students to give you a “thumbs up” if they know the product/answer. Have students whisper out the answer. Let the students know that you would like them to use pictures, words, numbers/symbols, etc. to show you ways they could use to solve the question. “How could you prove 56 is the correct answer?”

As the students are working, circulate the class noticing who may seem stuck. If a child seems unsure how to start, consider asking “Can you think of another way to say 8×7 ? If they do not think/say “Eight groups of seven” you may wish to say “Could we think of it as 8 groups of 7? Is there a way you could draw a picture to help you?” When you ask guiding questions, it is important to record this information on a sticky note so that when you are reviewing the strategies they used, you don't forget that you provided support.

Gather the students back together and ask them to share their strategies. Record these on a chart paper.

What do my students know about multiplication? And what misconceptions do they have?

Using the students' responses, you will carefully review each one and record which key understandings your students demonstrated. Just because a child did not demonstrate a key concept (e.g., skip counting) does not mean they do not understand this. Generally, you will be able to use your class list as a guide (e.g., 6 out of 28 students demonstrated an understanding that multiplication is connected to division) to know which concepts your students may need more time exploring.

What questions do my students have about multiplication?

What opportunities are there for integrating aboriginal perspectives?

Structured and Guided Inquiry Questions:

Seeing items in groups/sets: What comes in 2's, 3's and 4's (real world connections)

Inviting and inspiring materials:

- What Comes in 2's, 3's, and 4's by Suzanne Aker
- Chart Paper
- Markers

Guiding Questions:

When you look at humans, what parts of our body come in sets (e.g., eyes, fingers, toes)?

What numbers were easier to think about? Why might that be?

Which numbers were more challenging to come up with examples for?

How can you count your collection? Students need TONS of opportunities at this level counting in multiples using large quantities.

Inviting and inspiring materials:

- Loose materials in plastic bags (e.g., pasta pieces, buttons, pennies, glass stones, rocks, pompoms). Depending on your students you may want to have several different quantities available (e.g., 20 – 50 kit, 50 – 100 kit, 100 – 200, and 300 - 500 kit, and a kit with items that come in multiples like boxes of pencils that come in 12's)
- Tools to help students organize their count – including ice cube trays, cups, cupcake liners, ten frames, etc.

Guiding Questions:

Estimate how many are in your collection.

How can you count your collection?

Can you count it another way?

Can you record how you counted?

What happens when you count your collection another way? (e.g., Does the total change?)

I notice your finding it challenging to count by 4's... is there something you could use to help you keep track (e.g., hundred's chart and a highlighter)

Recording template/Helpful articles/resources: <http://www.meaningfulmathmoments.com/counting-collections.html>

How might number lines or hundreds charts help us to count sets of items using skip counting?

Inviting and inspiring materials:

- Completed brainstormed lists from the lesson What Comes in 2's, 3's, and 4's by Suzanne Aker
- Number Lines (0 – 100)
- Hundreds charts
- Markers

Guiding Questions:

Pick a multiplication question or roll two dice to create a “groups of” question.

How can you show one group of ____?

What is the pattern?

How might T-charts help us to count groups of items?

Inviting and inspiring materials:

- Completed brainstormed lists from the lesson What Comes in 2's, 3's, and 4's by Suzanne Aker
- T Charts
- Number Lines
- Markers
- Two of Everything by Lily Toy Hong – book explores doubling

Guiding Questions:

Pick an item from one of the lists. If you had 8 sets, how many might you have?

What do the numbers on the left-hand side of the T chart represent? And the right?

Do you notice a pattern?

How might the following materials help us think about multiplication as groups of?

Inviting and inspiring materials:

- Counters (e.g., pompoms, beans, feathers, mini erasers)
- Numbers (wooden, plastic, dice, foam, felt)
- Cuisenaire rods
- Base 10 blocks and/or ten frames
- Multiplication equations (e.g., $7 \times 3 =$) or dice

- Cups and cubes
- Paper for Circle and Stars <https://www.kyrene.org/cms/lib/AZ01001083/Centricity/Domain/1309/Circles and Stars Directions.pdf>
- Children's Literature books (The Best of Times, Grapes of Math, Amanda Bean's Amazing Dream, Too Many Frogs) Look at an illustration from the book. What question could you ask that multiplication would help you solve?
- Dim Sum for Everyone by Grace Lin – Your family goes out to dinner for Dim Sum. The dishes of Dim Sum are served with 3 treats in each bowl. How many treats would be served to your table? What if your family had 5 rounds of Dim Sum – how many treats would be served now?
- One Is A Snail, Ten Is A Crab by April Sayre and Jeff Sayre – What number of legs can you represent using multiplication?

Guiding Questions:

How did you represent the first factor? And the second factor?

What strategies can you use to determine how many altogether?

How do we assist students in understanding multiplication as arrays?

Inviting and inspiring materials:

- Representing multiplication using square tiles – How do squares and rectangles help you think about multiplication? What do you notice when you build arrays with the same factors (e.g., 7 x 7?)
- Cuisenaire rods
- Base 10 blocks
- Children's literature – My Full Moon is a Square by Elinor Pinczes – Using square tiles, how can we represent the arrays the flies created? How could we describe these? Can you represent the arrangement of ___ flies?
- Gold fish crackers – Fish like to swim in groups. How many equal groups can you make?
- Construction paper – Can you create/show some of arrays might you see at night in the city (E.g., windows on building)?
- The chocolate squares problem – Hershey's Milk Chocolate Multiplication book <http://investigatinglearning.blogspot.ca/2012/04/hersheys-chocolates-multiplication-fun.html>

Guiding Questions:

Tell me about your thinking... how did you decide how to arrange your squares?

What did you notice about the arrangements?

How could represent your question?

Are there other ways you could "read" your array?

When you look at your array can you think about how multiplication and division are related?

What happens when you multiply an even number x even number? Or odd number x odd number? Or an even number x odd number?

Inviting and inspiring materials:

- Screen-casting app
- Counters (e.g., pompoms, beans, feathers, mini erasers)
- Numbers (wooden, plastic, dice, foam, felt)

Guiding Questions:

Tell me about your thinking...

What patterns did you notice?

Do you think your reasoning is true in all situations?

How are multiplication and division related?

Inviting and inspiring materials:

- Counters (e.g., pompoms, beans, feathers, mini erasers)
- Numbers (wooden, plastic, dice, foam, felt)
- Square tiles
- Cuisenaire rods
- Children's literature books that divide items into equal groups. For example: 365 Penguins - This book is also great for exploring factors, and prime and composite numbers.

Guiding Questions:

Can you build me a model to explain your thinking?

Tell me about your thinking...

How can doubling help you with multiplication by four?

How does doubling and halving help you with multiplication questions you don't know?

Inviting and inspiring materials:

- Counters (e.g., pompoms, beans, feathers, mini erasers)
- Numbers (wooden, plastic, dice, foam, felt)
- Cuisenaire rods

- Base 10 blocks
- Bingo daubbers
- Paper and pencil
- Minnie's Diner by Dayle Ann Dodds – This is a great book that features doubling.

Guiding Questions:

What do you notice? What do you wonder?

Will what you are thinking apply in all situations?

How can you use facts you know to solve question you don't know?

Inviting and inspiring materials:

- Numbers (e.g., wooden, plastic, number tiles, paper numbers, stickers, dice)
- White boards
- Area square model templates
- Math Racks
- Virtual apps from the Math Learning Centre (the Number Rack and Partial Products Finder) <https://www.mathlearningcenter.org/resources/apps>
- Base 10 blocks
- Cuisenaire Rods
- Bingo Daubbers
- Counters
- Rich Tasks including Three Act Tasks by Graham Fletcher <https://docs.google.com/spreadsheets/d/1hc1ReIbdJZbEA3fO6DE457wu4AKOfi6BFxWLRBXO-bA/edit#gid=0>

Guiding Questions:

What is the largest number you can make? Smallest? A number in-between?

Describe the different numbers you made?

Can you order the numbers you made?

Journal Prompts:

When I think of multiplication, I see...

Today I represented a multiplication question using...

I'm proud of...

I'm thinking now...

I know...

This reminds me of...

A connection I have...

I noticed...

I tried...

I wonder...

Which children's literature books could be used as provocations:

Multiplication Books

- What Comes in 2's, 3's and 4's by Suzanne Aker
- 365 Penguins by Jean-Luc Fromental
- 100 Hungry Ants by Elinor Pincz
- Minnie's Diner by Dayle Ann Dodds
- Dim Sum for Everyone by Grace Lin
- Hershey's Multiplication by Jerry Pallotta
- Too Many Frogs by Ann Hassett and John Hassett
- One is a Snail, Ten is a Crab by April Sayre and Jeff Sayre
- My Full Moon Is A Square by Elinor Pinczes

Formative Assessment:

How will I document and communicate the students' learning?

- Photos and videos
- Student journals
- Performance based assessment
- Checklists – based on observations
- Anecdotal notes
- Conferences with individual students
- Screencasting apps such as ShowMe or Explain Everything

Summative Assessment:

Ask yourself, what would you students need to demonstrate that would show you they have met the learning standard(s) for multiplication? It could be a quiz/test but could it also be a project?

How will I continue to support my students' understanding of Number Concepts throughout the year?

- Number Talks Quick Image of Arrays – think about how you can help them develop fluency with derived facts (e.g., think 10 for 9)
- Number Talks with Multiplication questions – Example from the Teaching Channel <https://www.teachingchannel.org/videos/4th-5th-grade-number-talks>
- Mathematical Instructional Routines including Same/Different <http://www.meaningfulmathmoments.com/same-or-different.html> and True/False <http://www.meaningfulmathmoments.com/truefalse-routine.html>
- Weekly Counting Collections
- Math Learning Center Apps (Number Rack and Partial Product Finder) <https://www.mathlearningcenter.org/resources/apps>
- Reading and discussing any of the above books
- Rich Problems such as Three Act Tasks <https://gfletchy.com/3-act-lessons/>
- Open questions from Marian Small's books
- Paint Splat Arrays <https://reflectionsinthewhy.wordpress.com/2017/10/24/paint-splatter-arrays/>

- Independent Practice or with Partners Multiplication Games http://www.meaningfulmathmoments.com/uploads/1/1/1/9/11190716/multiplication_-_mastering_the_facts.pdf
 - Circles and Stars (Groups Of) <https://www.youcubed.org/tasks/circles-stars/>
 - Pepperoni Pizza (Groups Of) <https://www.youcubed.org/tasks/pepperoni-pizza/>
 - How Many Rows? How Many in Each Row? <https://www.youcubed.org/tasks/many-rows-many-row/>
 - How Close to 100? <https://www.youcubed.org/tasks/how-close-to-100/>
 - Matching Math Cards <https://www.youcubed.org/tasks/math-cards/>
 - Tic-Tac-Toe-Products (Practicing Known Facts) <https://www.youcubed.org/tasks/tic-tac-toe-products/>
 - Big Bad Wolf (Factors) <https://www.youcubed.org/tasks/big-bad-wolf/>